Broadband, Local Employment, and Population Growth*

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Abstract

I investigate the relationship between broadband, employment, and population growth to determine the causal directions between these processes. Starting from the framework of simultaneous employment-population models, I introduce a model to account for the potential simultaneous process of broadband-employment-population for regional economies. The model is furthered by spatial econometric techniques and a causal interpretation is given between these three dynamic processes. By establishing a causal direction, the claim that broadband spurs job growth is quantitatively evaluated as well as other competing hypotheses across the three processes. Results indicate that broadband deployment is an endogenous process and correcting for endogeneity fails to find support of claims that assert broadband growth leads to employment growth for a region. This paper illuminates the need for structural models involving broadband in evaluating regional economic impacts.

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1 Introduction

High-speed digital communication has become pervasive in most places throughout the United States. The rapidity and depth with which information and communications technology (ICT) infrastructure has penetrated across space and across a range of social strata, economic sectors and cultural milieux provide undeniable *prima facie* evidence of its value to society. Substantial direct investments of public funds have facilitated this process - including, most recently, $7.2 billion set aside for broadband deployment as part of the American Recovery and Reinvestment Act of 2009. Other public-private partnerships and quasi-governmental organizations (e.g., Connect America) similarly aim to facilitate expansion of the digital infrastructure.

Such large public investments inevitably merit investigation into their economic impacts. Proponents of programs aimed at facilitating the spread and penetration of broadband generally point to research projecting large aggregate economic benefits from widespread broadband deployment (Crandall et al., 2001, 2007; Greenstein and McDevitt, 2009). But an interesting aspect of public spending to promote broadband deployment - and one that studies of aggregate impacts do not really capture - is that a large proportion of that spending is place-based, i.e., targeting specific geographic locations. Indeed, broadband promotion programs are very commonly justified on the basis of projected *local* economic development in and around the target location.

A growing literature has found significant associations between various measures of broadband deployment and such indicators of local economic activity as the employment, number of firms, and average sales (Gillett et al., 2007; Stenberg et al., 2009; Osorio, 2006; Shideler et al., 2007). But there exists little, if any, evidence that convincingly demonstrates a significant causal relationship between an increase in broadband availability and an increase in economic activity (e.g. employment growth). Kolko (2012) ascribes three main deficiencies in the existing literature to date that need attention in order to establish such "predictive causality" (Diebold, 2007, p.201): (a) simultaneity in the determination of broadband provision, population and employment, with the attendant implications for inferences over the direction of causality between broadband and growth (of both firms and population); (b) spillover effects related to the movement of firms and households in response to the opportunities that high-speed digital communication might facilitate; and (c) heterogeneity in the magnitude and direction of impacts, both spatially and across economic sectors.

In this paper I take steps in the direction of addressing all three of these concerns in analyzing the link between broadband provision, population growth, and employment growth, a measure of local economic performance. Using county-level data over the period 2008-2012 from the 48 contiguous states, I exploit the predetermined nature of lagged variables to address endogeneity concerns. The empirical model's use of lagged values of population and employment also renders it consistent with underlying equilibrium adjustment models for both firms and residents developed by Steinnes (1977, 1982), and subsequently by Carlino and Mills (1987) in their influential examination of whether "jobs follow people" or "people follow jobs." I explicitly allow for spatial spillovers by incorporating spatial autoregressive processes within the econometric analysis. Finally, I conduct disaggregated analysis that assesses differential employment effects across different economic sectors.

The paper is laid out as follows. The next section, 2, reviews the previous literature on broadband impacts as well as causal analysis on population, employment, and infrastructure. Then, the conceptual framework for the relationships of broadband, employment, and population is introduced in section 3. Next, section 4 provides a description of the econometric strategy to identify the key elements of the causal relationships. Following an overview of data sources in section 5, econometric results are presented as well as their causal implications with section 6. In the final section, 7, I consider a variety of extensions to this analysis meriting future exploration.

2 Do "People Follow Jobs" or do "Jobs Follow People?"

Modeling the interrelationship between the demand and supply of labor/residents started with Steinnes and Fisher (1974) and subsequently furthered by Steinnes (1977, 1982). The model in Steinnes and Fisher (1974) links the employment and residence markets through two jointly determined equations that depart from the assumption in prior models that job growth is an exogenous process and causes population growth. By allowing for a system of simultaneous equations, Steinnes' model allows for testing the causal relationship between employment growth and population growth. The model assumes a static equilibrium, which is an arguably unrealistic assumption within the model as seen by temporal population shifts across the United States. Because of data availability concerns, the Steinnes studies utilized data at the Standard Metropolitan Statistical Areas (SMSA) level across at most 7 time periods. The sample data are of reasonably homogeneous units and are treated as a pooled data. Each study concludes that "jobs follow people." This implies that whereas population growth is a significant predictor in the employment equation, employment growth is not a significant predictor of population.

Mills and Price (1984) introduced an equilibrium adjustment variable to the preceding model, whereby employment and population levels adjust towards equilibrium over time – i.e. relaxing the assumption that the system is in a constant equilibrium across time. Carlino and Mills (1987) furthered this by lagging the explanatory variables in the model and by using data at the county level. Use of lags mitigates the potential simultaneity bias that may be present in a system in which variables are measured at an infrequent rate. The data improvements also changed the scope of research, as county level data are more heterogeneous and covers the entire United States. Neither study directly tested for causality in the employment and population relationship, although their reduced form models did allow for testing of a causal relationship.

Boarnet (1994) refocuses the employment-population dynamics on the SMSA level, specifically New Jersey, and accounts for spatial dynamics. In particular, none of the previous models accounts for the potential that labor markets extend across regional boundaries. Rather, prior analyses assumes that each unit of observation is independent of the other unit. A plausible argument is that employment in one sub-region brings population to the surrounding sub-regions. In practice, this manifests itself in the form of individuals commuting across boundary lines. To account for this, Boarnet defines a set of neighbors for each sub-region in his analysis based upon geographic distance from other sub-regions. These neighbors then make up the corresponding labor market or housing market that is included in the system of equations. After accounting for these spatial issues, Boarnet concludes that "jobs follow people" based on his New Jersey data.

Henry et al. (1997, 2001) furthers the Boarnet model to account for what he considers spread, or backwash, effects. The effect that Henry et al. accounts for is essentially whether growth in employment or population for one area affects its neighboring areas. If an increase in employment for location i simultaneously increases employment in neighboring locations j, then this is termed a spread effect. Conversely, if the increase in i reduces employment in neighboring locations j, then this is termed a backwash effect. The main interest that is derived from these models is to understand the relationship between urban growth and its surrounding rural areas. By looking at the growth rates for urban, urban fringe, and hinterland areas they test for spread or backwash effects. A spread effect would imply the hinterlands derive growth from urban areas while a backwash effect would be if growth in urban areas siphons resources away from the hinterlands.[1] This research introduces more sophisticated spatial econometric methods by introducing a spatially lagged dependent in the equations for both employment and population. Known as a spatial autoregressive model (SAR) within spatial econometrics, the sign of the parameter associated with the spatially

[1] The models developed by Henry aim to address the differential in growth rates of urban-core, urban-fringe, and rural areas by using urban growth rates as an interaction term. This strategy could be used to evaluate the claims that an urban-rural digital divide exists and if so, to what degree. This is left for future research.

lagged endogenous variable allows one to investigate whether spread or backwash effects are present. Their models generally support the "people follow jobs" hypothesis.

Bollinger and Ihlanfeldt (1997) establishes a framework to assess the roll of infrastructure in the population-employment nexus. Specifically, they use the Boarnet framework to look at the impact of MARTA, the mass-transit system of Atlanta, on the composition of employment and population from 1980 to 1990. They use census tract level data and the percentage of a tract within a quarter mile of a MARTA station as the variable of interest. Their model uses simultaneous equations of employment and population to account for the interrelated relationship between employment and population, as well as reduce any simultaneity bias. Their findings indicate that MARTA did not have a discernible impact on total population or employment, but it impacted the composition of employment in favor of the public sector. The decision of where MARTA stations located are treated as exogenous, and so their results may indicate a choice on the types of employment MARTA aimed to help rather than MARTA as the driver of employment changes for the city. Where broadband is a form of infrastructure, their work establishes a framework to determine the economic impact of infrastructure.

Kolko (2012) model is a subset of the Carlino and Mills (1987) model. Instead of focusing on the relationship between employment and population, Kolko focuses on employment and broadband at the zip code level. While Kolko does not model the broadband dispersion processes, he does use instrumental variable techniques in estimating the relationship between local employment growth and broadband deployment. Kolko argues that areas with uneven terrain increase the costs associated with deploying broadband for a region while being weakly correlated with employment growth. His reasoning implies that the average slope of terrain can be a weak instrument for broadband. Kolko finds that increases in broadband providers leads to increases in employment as a whole, as well within certain industries. However, Kolko cautions that while his findings are suggestive of a causal relationship, his analysis may suffer from endogeneity concerns insofar as average slope of terrain being correlated with employment growth.

Deller et al. (2001) evaluates the simultaneity of employment, population, and per capita income by extending the original Carlino and Mills (1987) to a system of three equations. The focus of the article was to evaluate the role of amenities on the quality of life for rural economic growth. While the model involves a system of three simultaneous equations, only the reduced form of the model is estimated. The results of the model indicate that natural amenities are positive determinants of employment, population, and per capita income for the 2,243 rural U.S. counties.

The literature on the causal direction of employment and population growth leads to a few natural extensions with broadband. For one, broadband may be an important factor that drives either employment or population growth. This would lead to including broadband deployment in the two equation model first introduced by Steinnes and Fisher (1974) in order to address whether or not "jobs follow broadband" or "people follow broadband." This is developed in section 3.1. The model also accounts for labor market effects for employment and population that are defined spatially *a la* Boarnet (1994).

I then consider in section 3.2, a more flexible structure capable of allowing for causality of broadband on employment or on population growth. This is to account for the process of broadband deployment in order to accommodate for the possibility that "broadband follows jobs" or "broadband follows people." Therefore, the model is extended to account for a process of broadband deployment and leads to a system of three simultaneous equations as in Deller et al. (2001). Because of concerns for the diffusion of broadband deployment following a spatial process, I also allow for a spatial lag with respect to broadband. The methods are described in the following section.

3 Conceptual Framework

3.1 Population-Employment Model

The Boarnet (1994) model describes an equilibrium relationship between population and employment location that depends on the surrounding area's level of population and employment. A general specification of this relationship can be written as:

$$P^*_{i,t} = f\left(\overline{E}^*_{i,t} \,\middle|\, \mathbf{Z}_{P_{i,0}}\right) \tag{1a}$$

$$E^*_{i,t} = g\left(\overline{P}^*_{i,t} \,\middle|\, \mathbf{Z}_{E_{i,0}}\right) \tag{1b}$$

where $P^*_{i,t}$ and $E^*_{i,t}$ are the equilibrium level of population and employment for location i at time t. The vectors $\mathbf{Z}_{P_{i,0}}$ and $\mathbf{Z}_{E_{i,0}}$ are initial conditions of variables within the models system that determine the equilibrium level. These variables are specific to population and employment, respectively. There is no restriction that variables in $\mathbf{Z}_{P_{i,0}}$ cannot appear in $\mathbf{Z}_{E_{i,0}}$ and in practice they do. However, in order to identify the system there must be at least one variable in $\mathbf{Z}_{P_{i,0}}$, and in $\mathbf{Z}_{E_{i,0}}$, that does not appear in the other.[2]

The variables $\overline{P}^*_{i,t}$ and $\overline{E}^*_{i,t}$ indicate the level of population and employment in the surrounding labor market areas of location i.[3] To define the labor market, a neighborhood set is needed. For general purposes, I shall define an $n \times n$ matrix $\mathbf{W}_l$ where n is the number of

[2] Boarnet (1994) uses transportation access and environmental amenities impacting residents or firms in his study. Additional variables included in the population equation but omitted from the employment equation were: percent black, percent hispanic, poverty rate, violent crime rate, proportion of housing stock before 1940, municipal public expenditures per capita, and per capita local tax payments. The variables included in the employment equation but omitted in the population equation were: commuter rail access, number of farm property parcels, per employee expenditures on streets and sewage, equalized property tax rate, labor market's total manufacturing employment, and total retail employment.

[3] Boarnet defined the labor market using an inverse distance function:

$$\overline{X}_i = X_i + \sum_{j \neq i} \frac{X_j}{d^{\alpha}_{ij}}$$

Where i denotes the focal region, j denotes all other regions, d_{ij} is the distance between the centroids in each region, and the parameter α describes how the housing or labor market relationship across regions dampens with distance. The subscript t has been suppressed because of redundancy.

units which are ordered 1 to n in both the columns and rows of the matrix and l refers to the equation of interest. Each element, $\mathbf{w}_{ij,l}$, denotes the relationship between observation i and j based upon a function of the spatial characteristics of the data. Typical functions are based on contiguity of areas, distance bands, or k-nearest neighbors depending on the characteristics of the geography and economic process of interest. Generally, this can be stated as:

$$\overline{P}_{i,t} = P_{i,t} + \phi_P \sum_{j \neq i} \mathbf{w}_{ij,P} P_{j,t} \tag{2a}$$

$$\overline{E}_{i,t} = E_{i,t} + \phi_E \sum_{j \neq i} \mathbf{w}_{ij,E} E_{j,t} \tag{2b}$$

where the parameters ϕ_P and ϕ_E represent the degree to which the location i is connected to its neighbors j in the housing and labor markets. This connection across labor and housing markets can loosely be interpreted as an economic cost for living in one region and working in a neighboring region. The spatial weight matrices, $\mathbf{W}_P$ and $\mathbf{W}_E$, need not be the same, i.e. reflecting labor markets and housing markets do not necessarily possess the same spatial characteristics.

Equations 2 have been written in summation form, but they can also be expressed in vector form to further illustrate the relationship:

$$\overline{P}_t = P_t + \phi_P \mathbf{W}_P P_t = \left(\mathbf{I}_n + \phi_P \mathbf{W}_P\right) P_t \tag{3a}$$

$$\overline{E}_t = E_t + \phi_E \mathbf{W}_E E_t = \left(\mathbf{I}_n + \phi_E \mathbf{W}_E\right) E_t \tag{3b}$$

where the variables P_t and E_t are stacked vectors of $P_{i,t}$ and $E_{i,t}$, and $\mathbf{I}_n$ is the $n \times n$ identity matrix.

If I assume a linear functional form for equation 1, the system can be paramaterized stochastically as follows:

$$P_{i,t}^{*} = \alpha_1 \mathbf{Z}_{P_{i,0}} + \alpha_2 \overline{E}_{i,t}^{*} + u_{i,t} \tag{4a}$$

$$E_{i,t}^{*} = \beta_1 \mathbf{Z}_{E_{i,0}} + \beta_2 \overline{P}_{i,t}^{*} + v_{i,t} \tag{4b}$$

where the vectors α_1 and β_1 indicate the marginal effects of control variables on population or employment, α_2 and β_2 are the parameters of interest in determining a causal relationship, with $u_{i,t}$ and $v_{i,t}$ as classical disturbance terms.[4]

The equilibrium levels of population and employment, $P_{i,t}^{*}$ and $E_{i,t}^{*}$, are unobservable. Following the approach introduced by Mills and Price (1984), I account for these variables through observables by assuming a process whereby actual levels for population and employment converge toward their equilibrium levels. This process can be described as:[5]

$$\Delta P_{i,t} = P_{i,t} - P_{i,t-1} = \lambda_P \left(P_{i,t}^{*} - P_{i,t-1} \right) \tag{5a}$$

$$\Delta E_{i,t} = E_{i,t} - E_{i,t-1} = \lambda_E \left(E_{i,t}^{*} - E_{i,t-1} \right) \tag{5b}$$

where P and E indicates the observable level of population and employment at the given location and time period. The parameters λ_P and λ_E are the rates of adjustment towards the equilibrium for population and employment. Further, equation 5 can be rearranged to calculate the rate of change for the population and employment levels as $(1-\lambda_P)$ and $(1-\lambda_E)$

[4] In order for this system to be identified, the vectors $Z_{P_{i,0}}$ and $Z_{E_{i,0}}$ must have at least on variable that is not included in the other vector. This is typically known as the exclusion restriction in the literature on systems of equations.

[5] There can be different functional forms within the model. For example, the equilibrium adjustment lag from equations 5 and 13 could potentially be of a different form:

$$\Delta X_{i,t} = log\left(\frac{X_{i,t}}{X_{i,t-1}}\right) = \lambda_X log\left(\frac{X_{i,t}^{*}}{X_{i,t-1}}\right)$$

Or that the general relationship assumed is non-linear. I am not as concerned about these partly because the literature has not considered other functional forms but also because these seem like pointless exercises in evaluating a functional form. I doubt that the added complexity will be a fruitful exercise for its associated cost.

respectively.[6]

The relationship can also be rewritten in terms of levels instead of differences as:

$$P^*_{i,t} = \frac{1}{\lambda_P} P_{i,t} + \left(1 - \frac{1}{\lambda_P}\right) P_{i,t-1} = \frac{1}{\lambda_P} P_{i,t} + \left(\frac{\lambda_P - 1}{\lambda_P}\right) P_{i,t-1} \tag{6a}$$

$$E^*_{i,t} = \frac{1}{\lambda_E} E_{i,t} + \left(1 - \frac{1}{\lambda_E}\right) E_{i,t-1} = \frac{1}{\lambda_E} E_{i,t} + \left(\frac{\lambda_E - 1}{\lambda_E}\right) E_{i,t-1} \tag{6b}$$

This rewritten equation allows estimation of the final equation with a different functional form, i.e. to help check if the parameter estimates are sensitive to the functional form of the adjustment lags.

The relationships in equation 5 can be substituted into equation 4 to yield:

$$\Delta P_{i,t} = \lambda_P \left(\alpha_1 \mathbf{Z}_{P_{i,0}} + \alpha_2 \overline{E}^*_{i,t}\right) - \lambda_P P_{i,t-1} + u_{i,t} \tag{7a}$$

$$\Delta E_{i,t} = \lambda_E \left(\beta_1 \mathbf{Z}_{E_{i,0}} + \beta_2 \overline{P}^*_{i,t}\right) - \lambda_E E_{i,t-1} + v_{i,t} \tag{7b}$$

There are two forces that drive the relationship between population and employment across equations. One is the cross-dependence of employment or population via α_2 and β_2. The other is from the equilibrium adjustment lags of λ_P and λ_E.[7] The adjustment parameters affect not only the rate at which population or employment adjusts towards its equilibrium level, but also the impact that a change in another explanatory variable affects the observable change in population or employment.

By using differences in observables and assuming an adjustment towards equilibrium, the model can be estimated with one more transformation. Although the equilibrium level for the labor market of employment or housing market of population is unobservable, I can use

[6] For the population equation, this can be seen by taking the derivative of $P_{i,t}$ with respect to $P_{i,t-1}$:

$$\frac{\partial P_{i,t}}{\partial P_{i,t-1}} = \lambda_P P^*_{i,t} + (1 - \lambda_P)$$

which represents the growth rate in population as well as its convergence towards equilibrium.

[7] See Dynamic System for details.

equations 2 and 5 in the defined labor market to rearrange the model to be of the form:

$$\Delta P_{i,t} = A_1 \mathbf{Z}_{P_{i,0}} + A_2 E_{i,t-1} + \Phi_E \mathbf{W}_E E_{i,t-1} + \frac{A_2}{\lambda_E} \Delta E_{i,t} + \frac{\Phi_E}{\lambda_E} \mathbf{W}_E \Delta E_{i,t} - \lambda_P P_{i,t-1} + u_{i,t} \quad \text{(8a)}$$

$$\Delta E_{i,t} = B_1 \mathbf{Z}_{E_{i,0}} + B_2 P_{i,t-1} + \Phi_P \mathbf{W}_P P_{i,t-1} + \frac{B_2}{\lambda_P} \Delta P_{i,t} + \frac{\Phi_P}{\lambda_P} \mathbf{W}_P \Delta P_{i,t} - \lambda_E E_{i,t-1} + v_{i,t} \quad \text{(8b)}$$

where the parameters have been redefined as $\Phi_E = A_2 \phi_E$, $\Phi_P = B_2 \phi_P$, $A_j = \alpha_j \lambda_P$ and $B_j = \beta_j \lambda_E$ for $j = 1, 2$. The model now includes only observable economic variables and can be estimated. The particular model is a system of two equations and can be estimated by accounting for the endogenous relationship that arises between $\Delta P_{i,t}$ and $\Delta E_{i,t}$ (due to simultaneity) and the labor and housing market effects from $\Delta \overline{E}_{i,t}$ and $\Delta \overline{P}_{i,t}$. An alternative, algebraically equivalent, form of the model can be written as:[8]

$$P_{i,t} = A_1 \mathbf{Z}_{P_{i,0}} + \frac{A_2}{\lambda_E} E_{i,t} + \frac{\Phi_E}{\lambda_E} \mathbf{W}_E E_{i,t} + \frac{A_2 (\lambda_E - 1)}{\lambda_E} E_{i,t-1} + \frac{\Phi_E (\lambda_E - 1)}{\lambda_E} \mathbf{W}_E E_{i,t-1} + (1 - \lambda_P) P_{i,t-1} + u_{i,t} \quad \text{(9a)}$$

$$E_{i,t} = B_1 \mathbf{Z}_{E_{i,0}} + \frac{B_2}{\lambda_P} P_{i,t} + \frac{\Phi_P}{\lambda_P} \mathbf{W}_P P_{i,t} + \frac{B_2 (\lambda_P - 1)}{\lambda_P} P_{i,t-1} + \frac{\Phi_P (\lambda_P - 1)}{\lambda_P} \mathbf{W}_P P_{i,t-1} + (1 - \lambda_E) E_{i,t-1} + v_{i,t} \quad \text{(9b)}$$

Boarnet (1994) used a spatial two-stage least squares estimator based on Anselin (1988) to

[8] In A.1.1, each equation of the model is solved in terms of only exogenous variables.

account for the spatial lag in the endogenous variables. This model collapses to Carlino and Mills (1987) if the labor market is not defined spatially, which would still require methods to deal with the simultaneity present in the model. This is due to the endogeneity of population and employment variables in the model and necessitates econometric methods to correct this bias. In the non-spatial models of Mills and Price (1984), Carlino and Mills (1987), and Kolko (2012), the typical method has been a two-stage estimation procedure.

Table 1: Necessary Conditions for Causality: Two Equation Model

	"people"	"jobs"	"broadband"
"people follow"	—	$\alpha_2 > 0$	$\alpha_{1,BB} > 0$
"jobs follow"	$\beta_2 > 0$	—	$\beta_{1,BB} > 0$

In order to evaluate how infrastructure such as broadband affects population and employment, I can promote the broadband variable from $\mathbf{Z}_{P_{i,0}}$ and $\mathbf{Z}_{E_{i,0}}$ in each equation and define as $A_{1,BB}$ and $B_{1,BB}$. This was first suggested in Bollinger and Ihlanfeldt (1997) with respect to the infrastructure of mass transit for Atlanta. Evaluation of the parameters associated with the initial conditions of broadband in each equation, as seen in table 1, allows for inference on whether "jobs follow broadband" or "people follow broadband." If broadband is a significant predictor in the employment (population) equation, then this is consistent with "jobs" ("people") "follow broadband." Asserting these claims would be valid under the assumption that broadband deployment is an exogenous process. The assumption is fairly heroic, and so I adopt Kolko (2012) method of instrumenting broadband with average slope of the terrain. While this is an attractive model that can be useful in evaluating certain types of infrastructure, this lacks insight on where broadband is deployed. This is expanded further in the next section.

3.2 Extended Model for Broadband

Following the Boarnet (1994) spatial model of population and employment, I can conceptualize a three-way relationship with no *a priori* causal relationship between broadband, population, and employment growth. A non-spatial model with a system of three simultaneous equations was first presented in Deller et al. (2001)

$$P^*_{i,t} = f\left(\overline{E}^*_{i,t}, \overline{BB}^*_{i,t} \;\middle|\; \mathbf{Z}_{P_{i,0}}\right) \tag{10a}$$

$$E^*_{i,t} = g\left(\overline{P}^*_{i,t}, \overline{BB}^*_{i,t} \;\middle|\; \mathbf{Z}_{E_{i,0}}\right) \tag{10b}$$

$$BB^*_{i,t} = h\left(\overline{P}^*_{i,t}, \overline{E}^*_{i,t} \;\middle|\; \mathbf{Z}_{B_{i,0}}\right) \tag{10c}$$

where the vectors $\mathbf{Z}_{P_{i,0}}$, $\mathbf{Z}_{E_{i,0}}$, and $\mathbf{Z}_{B_{i,0}}$ contain a set of initial conditions for variables specific to either population, employment, or broadband. As per the Population-Employment Model, in order for the system to be identified and a causal analysis carried out there must be at least one variable that is unique to each of the vectors.

Further, $\overline{BB}^*_{i,t}$ is the equilibrium level of the regional market for broadband. The market for broadband is defined in a similar manner of equation 2:

$$\overline{BB}_{i,t} = BB_{i,t} + \phi_B \sum_{j \neq i} \mathbf{w}_{ij,B} BB_{i,t} \tag{11a}$$

$$\overline{BB}_t = BB_t + \phi_B \mathbf{W}_B BB_t = \left(\mathbf{I}_n + \phi_B \mathbf{W}_B\right) BB_t \tag{11b}$$

As before, this specification allows for a spatially dependent relationship among the dependent variables. The inclusion of $BB^*_{i,t}$ in equations 10 allows for the possibility of an endogenous process for broadband deployments and allows for a causal relationship between all three of these variables. Policy makers who are interested in using broadband as a way to attract firms or high-skilled workers to an area would assert that a region's level of broadband is a significant, and positive, determinant of employment growth and or population growth. Testing this hypothesis also allows us to evaluate whether the positive relationship between broadband deployment and employment growth is because broadband is deployed in areas already experiencing, or expecting, large amounts of employment growth. This argument also applies to population growth. If this critique has any bite to it, then I would find that employment and population growth are positive and significant predictors of broadband growth in the broadband equation. Given a functional form for the relationship in equation 10, estimating these simultaneous equations enables testing this hypothesis.

Table 2: Necessary Conditions for Causality: Extended Model for Broadband

	"people"	"jobs"	"broadband"
"people follow"	—	$\alpha_2 > 0$	$\alpha_3 > 0$
"jobs follow"	$\beta_2 > 0$	—	$\beta_3 > 0$
"broadband follows"	$\gamma_2 > 0$	$\gamma_3 > 0$	—

As before, I assume a linear functional form for the processes by which population, employment, and broadband equilibrium levels are determined:

$$P^*_{i,t} = \alpha_1 \mathbf{Z}_{P_{i,0}} + \alpha_2 \overline{E}^*_{i,t} + \alpha_3 \overline{BB}^*_{i,t} + u_{i,t} \tag{12a}$$

$$E^*_{i,t} = \beta_1 \mathbf{Z}_{E_{i,0}} + \beta_2 \overline{P}^*_{i,t} + \beta_3 \overline{BB}^*_{i,t} + v_{i,t} \tag{12b}$$

$$BB^*_{i,t} = \gamma_1 \mathbf{Z}_{B_{i,0}} + \gamma_2 \overline{P}^*_{i,t} + \gamma_3 \overline{E}^*_{i,t} + \omega_{i,t} \tag{12c}$$

where $u_{i,t}$, $v_{i,t}$, and $\omega_{i,t}$ are classical error terms. Given this assumed functional form, it becomes clear that there are necessary conditions for a causal relationship between the dependent variables as summarized in table 2. These lead to hypothesized tests that can be carried out empirically and are discussed in Section 4. I assume that population, employment, and broadband are unrelated processes unless the data provide strong evidence of a relationship, tested under the model's implications.

Further, I assume that the level of broadband for a region over time adjusts toward an equilibrium level. This is the same as I assumed before for population and employment in equation 5:

$$\Delta BB_{i,t} = BB_{i,t} - BB_{i,t-1} = \lambda_B \left(BB^*_{i,t} - BB_{i,t-1} \right) \tag{13}$$

which can also be rewritten in the form:

$$BB^*_{i,t} = \frac{1}{\lambda_B} BB_{i,t} + \left(1 - \frac{1}{\lambda_B} \right) BB_{i,t-1} = \frac{1}{\lambda_B} BB_{i,t} + \left(\frac{\lambda_B - 1}{\lambda_B} \right) B_{i,t-1} \tag{14}$$

It is important to discuss the implications of such an equilibrium adjustment here. Infrastructure operates in a different manner than individuals or firms. A crucial difference is that individuals and firms have the ability to migrate, whereas it is extremely costly for infrastructure to move. By looking at the population and employment equilibrium adjustment from equations 5, the adjustment parameters λ_P and λ_E are an approximation to an unknown functional form of how these two variables evolve over time and across markets. The more mobile the factors are, the more accurate the equilibrium adjustment parameters approximate the true relationship.

Infrastructure is clearly not as mobile as population or employment. Infrastructure does have upkeep due to deterioration, which involves maintenance costs, of which failure to pay may cause the infrastructure to cease to function. Further, improvement in technology may make the current infrastructure obsolete. Insofar as broadband is a type of infrastructure, municipalities and telecommunication companies can choose an amount of capital to invest in broadband infrastructure. This investment decision may have a large upfront cost, but as technology progresses, these costs decline, which induces more areas to adopt. Further, the upfront costs differ across regions for multiple reasons. If there are certain types of existing telecommunication infrastructure[9] already in place, the costs may be substantially lower. The costs also differ based upon the type of terrain. Mountainous areas and areas with uneven landscapes involve more labor and capital to deploy, thereby increasing the costs of broadband. The characteristics of the housing stock also affect the costs of deployment. Broadband is expensive to install in older homes as they may not be currently wired to obtain access. Conversely, areas with increasing housing start-ups will face lower costs of deployment as broadband can be installed with other infrastructure involved in the building process. These factors suggest significant variation across both time and space, and hence an equilibrium adjustment of the form in equation 13 is still appropriate even though broadband is not as mobile as population or employment.

Substituting in equations 12 for the equilibrium levels in equations 5 and 13 gives us:

$$\Delta P_{i,t} = \lambda_P \left(\alpha_1 \mathbf{Z}_{P_{i,0}} + \alpha_2 \overline{E}^*_{i,t} + \alpha_3 \overline{BB}^*_{i,t}\right) - \lambda_P P_{i,t-1} + u_{i,t} \tag{15a}$$

$$\Delta E_{i,t} = \lambda_E \left(\beta_1 \mathbf{Z}_{E_{i,0}} + \beta_2 \overline{P}^*_{i,t} + \beta_3 \overline{BB}^*_{i,t}\right) - \lambda_E E_{i,t-1} + v_{i,t} \tag{15b}$$

$$\Delta BB_{i,t} = \lambda_B \left(\gamma_1 \mathbf{Z}_{B_{i,0}} + \gamma_2 \overline{P}^*_{i,t} + \gamma_3 \overline{E}^*_{i,t}\right) - \lambda_B BB_{i,t-1} + \omega i, t \tag{15c}$$

The system here mimics the system with only population and employment in that there are two forces that drive the above relationship. The cross-dependence terms of α_2, α_3, β_2, β_3, γ_2, and γ_3 all relate to the hypothesis of whether a process depends upon the other. Further, the equilibrium adjustment lag parameters λ_P, λ_E, and λ_B interact with the other variables in each equation in the same manner as the Population-Employment Model.

In the two-equations system between population and employment, the variables $\overline{P}_{i,t}$ and $\overline{E}_{i,t}$ represent the labor market levels of population or employment for a given area. The reasoning for the population equation is that residents choose their location based upon

[9] While fiber optic broadband access requires new fiber to be installed from the local exchange to the subscriber, other forms of broadband can use existing networks. Cable modem systems use existing hybrid fiber-coax Cable TV networks. xDSL systems use the twisted copper pair traditionally used for voice services by the plain old telephone system. Broadband powerline broadband technology uses the power lines feeding into the subscriber's home to carry broadband signals (Corning, 2005).

not only the available employment opportunities in a given area also but, from the larger commuting zone. Similar reasoning applies for the employment equation; a firm will look at the larger commuting zone in making its location decision to form or expand. For broadband, it is unclear if this process also exists as broadband access for a neighbor does not yield direct benefits. However, broadband access for neighbors could indicate future expansion and increases in broadband access for which firms or individuals can capture lower prices in anticipation.

I can substitute in for the endogenous variables for an estimable system:

$$\Delta P_{i,t} = A_1 \mathbf{Z}_{P_{i,0}} + A_2 E_{i,t-1} + \Phi_{E,A}\mathbf{W}_E E_{i,t-1} + \frac{A_2}{\lambda_E}\Delta E_{i,t} + \frac{\Phi_{E,A}}{\lambda_E}\mathbf{W}_E \Delta E_{i,t} + A_3 BB_{i,t-1} + \Phi_{B,A}\mathbf{W}_B BB_{i,t-1} + \frac{A_3}{\lambda_B}\Delta BB_{i,t} + \frac{\Phi_{B,A}}{\lambda_B}\mathbf{W}_B \Delta BB_{i,t} - \lambda_P P_{i,t-1} + u_{i,t} \quad (16a)$$

$$\Delta E_{i,t} = B_1 \mathbf{Z}_{E_{i,0}} + B_2 P_{i,t-1} + \Phi_{P,B}\mathbf{W}_P P_{i,t-1} + \frac{B_2}{\lambda_P}\Delta P_{i,t} + \frac{\Phi_{P,B}}{\lambda_P}\mathbf{W}_P \Delta P_{i,t} + B_3 BB_{i,t-1} + \Phi_{B,B}\mathbf{W}_B BB_{i,t-1} + \frac{B_3}{\lambda_B}\Delta BB_{i,t} + \frac{\Phi_{B,B}}{\lambda_B}\mathbf{W}_B \Delta BB_{i,t} - \lambda_E E_{i,t-1} + v_{i,t} \quad (16b)$$

$$\Delta BB_{i,t} = \Gamma_1 \mathbf{Z}_{B_{i,0}} + \Gamma_2 P_{i,t-1} + \Phi_{P,\Gamma}\mathbf{W}_P P_{i,t-1} + \frac{\Gamma_2}{\lambda_P}\Delta P_{i,t} + \frac{\Phi_{P,\Gamma}}{\lambda_P}\mathbf{W}_P \Delta P_{i,t} + \Gamma_3 E_{i,t-1} + \Phi_{E,\Gamma}\mathbf{W}_E E_{i,t-1} + \frac{\Gamma_3}{\lambda_E}\Delta E_{i,t} + \frac{\Phi_{E,\Gamma}}{\lambda_E}\mathbf{W}_E \Delta E_{i,t} - \lambda_B BB_{i,t-1} + \omega_{i,t} \quad (16c)$$

The parameters are redefined as $\Phi_{E,A} = A_2\phi_{EA}$, $\Phi_{B,A} = A_3\phi_{BA}$, $\Phi_{P,B} = B_2\phi_{PB}$, $\Phi_{B,B} = B_3\phi_{BB}$, $\Phi_{P,\Gamma} = \Gamma_2\phi_{P\Gamma}$, $\Phi_{E,\Gamma} = \Gamma_3\phi_{E\Gamma}$, $A_j = \alpha_j\lambda_P$, $B_j = \beta_j\lambda_E$, and $\Gamma_j = \gamma_j\lambda_B$ for $j = 1, 2, 3$. This larger model nests the Population-Employment Model of 8 under the condition that $\gamma_1 =$

$\gamma_2 = \gamma_3 = \phi_B = \alpha_3 = \beta_3 = 0$ so that the broadband process is exogenous to the system.[10] This allows for the larger system from equation 16 first to be estimated unrestricted and then to impose restrictions testing the relationship between the three endogenous variables.

The larger system can also be rewritten in the form:

$$P_{i,t} = A_1 \mathbf{Z}_{P_{i,0}} + \frac{A_2}{\lambda_E} E_{i,t} + \frac{\Phi_E}{\lambda_E} \mathbf{W}_E E_{i,t} + \frac{A_2 (\lambda_E - 1)}{\lambda_E} E_{i,t-1} + \frac{\Phi_E (\lambda_E - 1)}{\lambda_E} \mathbf{W}_E E_{i,t-1} + \frac{A_3}{\lambda_B} BB_{i,t} + \frac{\Phi_B}{\lambda_B} \mathbf{W}_B BB_{i,t} + \frac{A_3 (\lambda_B - 1)}{\lambda_B} BB_{i,t-1} + \frac{\Phi_B (\lambda_B - 1)}{\lambda_B} \mathbf{W}_B BB_{i,t-1} + (1 - \lambda_P) P_{i,t-1} + u_{i,t} \tag{17a}$$

$$E_{i,t} = B_1 \mathbf{Z}_{E_{i,0}} + \frac{B_2}{\lambda_P} P_{i,t} + \frac{\Phi_P}{\lambda_P} \mathbf{W}_P P_{i,t} + \frac{B_2 (\lambda_P - 1)}{\lambda_P} P_{i,t-1} + \frac{\Phi_P (\lambda_P - 1)}{\lambda_P} \mathbf{W}_P P_{i,t-1} + \frac{B_3}{\lambda_B} BB_{i,t} + \frac{\Phi_B}{\lambda_B} \mathbf{W}_B BB_{i,t} + \frac{B_3 (\lambda_B - 1)}{\lambda_B} BB_{i,t-1} + \frac{\Phi_B (\lambda_B - 1)}{\lambda_B} \mathbf{W}_B BB_{i,t-1} + (1 - \lambda_E) E_{i,t-1} + v_{i,t} \tag{17b}$$

$$BB_{i,t} = \Gamma_1 \mathbf{Z}_{B_{i,0}} + \frac{\Gamma_2}{\lambda_P} P_{i,t} + \frac{\Phi_P}{\lambda_P} \mathbf{W}_P P_{i,t} + \frac{\Gamma_2 (\lambda_P - 1)}{\lambda_P} P_{i,t-1} + \frac{\Phi_P (\lambda_P - 1)}{\lambda_P} \mathbf{W}_P P_{i,t-1} + \frac{\Gamma_3}{\lambda_E} E_{i,t} + \frac{\Phi_E}{\lambda_E} \mathbf{W}_E E_{i,t} + \frac{\Gamma_3 (\lambda_E - 1)}{\lambda_E} E_{i,t-1} + \frac{\Phi_E (\lambda_E - 1)}{\lambda_E} \mathbf{W}_E E_{i,t-1} + (1 - \lambda_B) BB_{i,t-1} + \omega_{i,t} \tag{17c}$$

The three-equation system is more flexible than the two-equation system, but there is a concern that the three equation system may not be necessary. Excessive modeling (i.e. treating broadband as endogenous when it is not) can lead to imprecise standard errors with associated variables of interest. If broadband is not in fact endogenous, then the Population-Employment Model is the relevant one for robustness checks for the types of jobs or people that may be attracted by broadband deployment. However, if the process is endogenous, then robustness checks will also focus on what types of areas are more likely to attract

[10] See appendix A.2 for details.

broadband as a way to understand what types of economies benefit most from broadband.

4 Econometric Strategy

4.1 System Typologies

The two models that are identified in equations 8 and 16 represent different approaches to modeling simultaneity, as discussed in Rey and Boarnet (2004). Rey and Boarnet lay out a taxonomy of simultaneous equation models that describe simultaneity through what they term feedback simultaneity or spatial simultaneity. Spatial simultaneity can arise from a spatial cross-regressive or spatial autoregressive process. Traditional simultaneous equations models contain what Rey and Boarnet described as feedback simultaneity, which results from an endogenous variable appearing on the right hand side of an equation. Because all simultaneity involves some sort of feedback mechanism, I refer to this as traditional feedback or traditional simultaneity. Equation 8 includes traditional feedback effects through the interconnection of ΔP and ΔE while equation 16 contains these feedbacks as well as including feedback through the inclusion of ΔBB. The presence of traditional simultaneity within the population and employment has been known since Steinnes and Fisher (1974) and the typical methods for correcting for this in estimation is some sort of a two-stage least squares procedure or maximum likelihood estimation.

A non-spatial simultaneous equation model takes the form of:

$$Y_1 = \alpha_0 + \alpha_1 Y_2 + \varepsilon_1 \tag{18a}$$

$$Y_2 = \beta_0 + \beta_1 Y_1 + \varepsilon_2 \tag{18b}$$

Because each equation is a function of the other, a simple ordinary least squares procedure on a single equation would have α_2 (β_2) correlated with its error term, and hence the estimated coefficients would be biased. To address this issue, models of this sort are typically solved via a multiple staged estimation approach or maximum likelihood methods.

4.1.1 Spatial Cross-Regressive Simultaneity

Spatial cross-regressive simultaneity typically appears along with feedback and represents a roughly comparable problem. A spatial cross-regressive model takes the following form:

$$Y_1 = \alpha_0 + \alpha_1 Y_2 + \alpha_2 W_2 Y_2 + \varepsilon_1 \tag{19a}$$

$$Y_2 = \beta_0 + \beta_1 Y_1 + \beta_2 W_1 Y_1 + \varepsilon_2 \tag{19b}$$

where W_i is an $n \times n$ matrix that defines the neighborhood set between Y_i and Y_j. The terms α_2 and β_2 serve as the spatial feedback from the neighboring values of the endogenous variables (e.g. labor market characteristic as in Boarnet (1994)). The purpose of this particular spatial feedback is to take increase the channels of which two endogenous explanatory variables are spatially connected. Boarnet originally applied this logic to the relationship between employment and population, using a spatially defined weighted average of population in the employment equation and a spatially defined weighted average of employment in the population equation. The intuition is that households will locate within a region near their place of employment but not necessarily within the boundaries of the region where the place of employment is. The same goes for where a firm might locate, their decision is not limited to population in a certain jurisdiction but includes the surrounding areas from which people can feasibly commute.

There are a few options for the estimation structure for this particular typology. If the theoretical underpinning of the spatial relationship between Y_1 and Y_2 is ambiguous, or allows for the possibility that there may not be a spatial relationship, then equation 19 can be estimated in the current form with emphasis on whether or not $\alpha_2 = 0$ as well as $\beta_2 = 0$. An unrestricted regression is appealing if the main interest is to answer whether or not a labor market relationship exists.

However, it is typically the case that the structural relationship from which equation 19 is derived will have restrictions in some form that generally imply $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$. If this is the case, then there are two choices for the econometrician: to test if this relationship holds, or to impose the relationship *a priori*.

4.1.2 Spatial Autoregressive Simultaneity

The other typology class that Rey and Boarnet (2004) describes is a spatially lagged dependent variable within each equation. This relationship is typically referred to as a spatial autoregressive (SAR) model in the spatial econometrics literature. A general form of this can be described as:

$$Y_1 = \alpha_0 + \alpha_1 Y_2 + \alpha_2 W_1 Y_1 + \varepsilon_1 \tag{20a}$$

$$Y_2 = \beta_0 + \beta_1 Y_1 + \beta_2 W_2 Y_2 + \varepsilon_2 \tag{20b}$$

The general intuition behind spatially lagging the dependent variable here is to account for potential spread or backwash effects in the dependent variable of interest (as in Henry et al. (2001)). In the context of an employment equation, this means that when there is a rise in the level of employment of location i, then its neighbors in location j will experience either an increase in their level of employment ($\alpha_2 > 0$, spread effect) or a decrease in their level of employment ($\alpha_2 < 0$, backwash effect). The causes for the increase or decrease stem from an empirical application to theory that either agglomeration effects exist or that consolidation of economic activity is occurring. This issue was first applied by Henry et al. (1997) with particular focus on the impacts of urban growth on rural areas.

If this feedback mechanism exists in a system of simultaneous equations and is not accounted for in estimation procedures, then the estimators will be biased. Anselin (1988) shows that ordinary least squares estimates for a SAR process are also biased and so different estimators need to be applied.

There may be cases where economic theory suggests a SAR process within a system of simultaneous equations. Other times, it may be the case that these spread or backwash effects existence within the relationship is the question of interest. Similar to the spatial cross-regressive effects, introducing a SAR process (as in equation 20) or testing whether the spatial relationship exists will imply different strategies for estimation. A theory driven approach is necessary to verify if a SAR process needs to be accounted for. If theory suggests that the relationship exists, then the system of equations needs to be estimated that accounts for the spatial relationship. These models can employ instrumental variable estimators (Kelejian and Prucha, 2004), maximum likelihood estimators, generalized method of moments estimators, or through Bayesian methods. If the spatial relationship in equation 20 needs to be tested but not imposed, one might consider estimating the system with some of the methods mentioned previously and then testing whether or not $\alpha_2 = 0$ and or $\beta_2 = 0$.

Finally, it bears mention that all three forms of simultaneity may be present and it can be the case that specific equations within the system exhibit some of these classes while the other equations do not. With a system of two equations, Rey and Boarnet (2004) define 35 unique typologies involving the three characteristics of traditional feedback, spatial cross-regressive, and spatial lag. The number of unique typologies increases with the number of equations involved in a system. It is not within the scope of this paper to classify all the typologies of the relationship between broadband, employment, and population. The purpose of this paper is to test for the causal relationship between the three economic processes and to use the insight of how these feedback mechanisms affect the estimation procedure and interpretation of results. My approach to doing so is discussed in the next section.

4.2 Estimation of Spatial System

4.2.1 Notation for System

I follow Kelejian and Prucha (2004) in using full information generalized spatial three stage least squares estimators to estimate equations 8 and 16. Use of these estimators potentially allows for causal inference on the relationship between broadband, employment, and population growth. Under certain assumptions, the estimators are consistent and asymptotically normally distributed. The proofs are beyond the scope of this paper, but can be found in Kelejian and Prucha (2004). The methods for these estimators are described and applied to equations 8 and 16 below.

In order to maintain notational consistency with Kelejian, I define the following system of n cross sectional units:

$$Y_n = Y_n B + X_n C + \overline{Y}_n \Lambda + U_n \tag{21}$$

with the vectors defined as:

$$\begin{aligned}
Y_n &= (y_{1,n}, \ldots, y_{m,n}) \\
X_n &= (x_{1,n}, \ldots, x_{m,n}) \\
U_n &= (u_{1,n}, \ldots, u_{m,n}) \\
\overline{Y}_n &= (\overline{y}_{1,n}, \ldots, \overline{y}_{m,n}) \\
W_n &= (W_{1,n}, \ldots, W_{m,n}) \\
\overline{y}_{j,n} &= W_n y_{j,n}, \;\; j = 1, \ldots, m
\end{aligned}$$

where $y_{j,n}$ is the $n \times 1$ vector of cross sectional observations of the dependent variable in the j-th equation; $x_{l,n}$ is the $n \times l$ vector of cross sectional observations of the l-th exogenous variable; $u_{j,n}$ is the $n \times 1$ disturbance vector in the j-th equation; W_n are $n \times n$ spatial weighting matrices of known constants, and B, C, and Λ are correspondingly defined parameter matrices of dimension $m \times m$, $k \times m$, and $m \times m$. The vector $\overline{y}_{j,n}$ is referred to as a spatial lag of $y_{j,n}$. The system contains m equations within it.

In addition, I allow for spatial autocorrelation in the disturbances of the following form:

$$U_n = \overline{U}_n R + E_n \tag{22}$$

with the vectors defined as:

$$\begin{aligned} E_n &= (\varepsilon_{1,n}, \ldots, \varepsilon_{m,n}) \\ R &= diag_{j=1}^{m}(\rho_j) \\ \overline{U}_n &= (\overline{u}_{1,n}, \ldots, \overline{u}_{m,n}) \\ \overline{u}_{j,n} &= W_n u_{j,n}, \;\; j = 1, \ldots, m \end{aligned}$$

where $\varepsilon_{j,n}$ denotes the $n \times 1$ vector of errors and ρ_j the spatial autoregressive parameter in the j-ith equation. The vector $\overline{u}_{j,n}$ is typically referred to as the spatial lag of $u_{j,n}$. Within this system, it is assumed that the mean of the error terms for each equation are zero, each equation's error has its own variance, and that the errors across equations are allowed to covary:

$$E[\varepsilon_{j,n}] = 0, \;\; E\left[\varepsilon_{j,n}\varepsilon'_{k,n}\right] = \sigma_{jk} I_n \tag{23}$$

Using compact notation, let $Z_{j,n} = \left(Y_{j,n}, X_{j,n}, \overline{Y}_{j,n}\right)$ denote the matrix of observations of right hand side variables that appear in the j-th equation, and let $\delta_j = \left(\beta'_j, \gamma'_j, \lambda'_j\right)'$ denote the corresponding parameter vector. I can then rewrite the j-th equation in equations 21 and 22 as:

$$\begin{aligned} y_{j,n} &= Z_{j,n}\delta_j + u_{j,n} \\ u_{j,n} &= \rho_j W_n u_{j,n} + \varepsilon_{j,n} \end{aligned} \tag{24}$$

Further, applying a Cochrane-Orcutt[11] transformation to equation 24 in order to account for spatial dependence in errors yields:

$$y^*_{j,n}(\rho_j) = Z^*_{j,n}(\rho_j)\,\delta_j + \varepsilon_{j,n} \tag{25}$$

where

$$y^*_{j,n}(\rho_j) = y_{j,n} - \rho_j W_n y_{j,n}$$
$$Z^*_{j,n}(\rho_j) = Z_{j,n} = \rho_j W_n Z_{j,n}$$

Stacking the equations from 25 yields

$$y^*_n(\rho) = Z^*_n(\rho)\,\delta + \varepsilon_n \tag{26}$$

where

$$y^*_n(\rho) = \left(y^*_{1,n}(\rho_1)', \ldots, y^*_{m,n}(\rho_m)'\right)'$$
$$Z^*_n(\rho) = diag^m_{j=1}\left(Z^*_{j,n}(\rho_j)\right)$$
$$\varepsilon_n(\rho) = \left(\varepsilon'_{1,n}, \ldots, \varepsilon'_{m,n}\right)'$$

and $\rho = (\rho_1, \ldots, \rho_m)'$ and $\delta = (\delta'_1, \ldots, \delta'_m)'$. Clearly, the residuals can be formed from $E\varepsilon_n\varepsilon'_n = \Sigma \otimes I_n$ where $\Sigma = (\sigma_{jk})$. While this implies homoskedasticity, the results generalize for any consistent variance-covariance matrix.[12]

[11] Cochrane and Orcutt (1949) details how this method applies to time-series data. However, Kelejian and Prucha (1997) extended this time-series technique to spatial methods.

[12] Kelejian and Prucha (2007) provides a framework for a non-parametric heteroscedasticity and autocorrelation consistent (HAC) estimator of the variancecovariance (VC) matrix for a spatial models which is utilized for robust standard errors.

4.2.2 Estimation Procedure

To gain consistent estimation of the general form of a spatial system of simultaneous equations as shown in equation 26, the first step is to estimate the model parameters δ_j from 24 by using a two-stage least squares estimator to each of the m equations:

$$\widetilde{\delta}_{j,n} = \left(\widetilde{Z}'_{j,n} Z_{j,n}\right)^{-1} \widetilde{Z}'_{j,n} y_{j,n} \tag{27}$$

where

$$\begin{aligned}
\widetilde{Z}_{j,n} &= P_H Z_{j,n} = \left(\widetilde{Y}_{j,n}, X_{j,n}, \widetilde{\overline{Y}}_{j,n}\right) \\
\widetilde{Y}_{j,n} &= P_H Y_{j,n} \\
\widetilde{\overline{Y}}_{j,n} &= P_H \overline{Y}_{j,n} \\
P_H &= H_n \left(H'_n H_n\right)^{-1} H'_n
\end{aligned}$$

and where H_n is the matrix of instruments which is formed as a subset of linearly independent columns of $(X_n, W_n X_n, W_n^2 X_n, \ldots)$. Based on $\widetilde{\delta}_{j,n}$, I can compute two-stage least squares residuals:

$$\widetilde{u}_{j,n} = y_{j,n} - Z_{j,n} \widetilde{\delta}_{j,n} \tag{28}$$

Once the two-stage least squares estimators are calculated for each equation, the residuals can be tested for spatial dependence with a Moran's I test proposed in Kelejian and Prucha (2001). Potentially, spatial autocorrelation could be due to some factor that is not accounted for that varies spatially. While failing to correct for this autocorrelation does not bias the estimators, this does affect the standard errors which in turn would affect inference on the structural parameters of interest.

To correct for spatial dependence in the error term, spatial autoregressive parameters

ρ_j are estimated using the residuals obtained via the first step. A generalized method of moments estimator proposed by Kelejian and Prucha (1999) is used for estimators of ρ_j and σ_{jj}. The estimators, defined as $\widetilde{\rho}_j$ and $\widetilde{\sigma}_{jj}$, are the nonlinear least squares estimators that minimize:

$$\left[g_{j,n} - G_{j,n}\begin{bmatrix}\rho_j \\ \rho_j^2 \\ \sigma_{jj}\end{bmatrix}\right]'\left[g_{j,n} - G_{j,n}\begin{bmatrix}\rho_j \\ \rho_j^2 \\ \sigma_{jj}\end{bmatrix}\right]$$

where

$$G_{j,n} = \frac{1}{n}\begin{bmatrix} 2\widetilde{u}'_{j,n}\widetilde{\overline{u}}_{j,n} & -\widetilde{\overline{u}}'_{j,n}\widetilde{\overline{u}}_{j,n} & n \\ 2\widetilde{\overline{u}}'_{j,n}\widetilde{\overline{\overline{u}}}_{j,n} & -\widetilde{\overline{\overline{u}}}'_{j,n}\widetilde{\overline{\overline{u}}}_{j,n} & Tr\left(W_n'W_n\right) \\ \widetilde{u}'_{j,n}\widetilde{\overline{\overline{u}}}_{j,n} + \widetilde{\overline{u}}'_{j,n}\widetilde{\overline{u}}_{j,n} & -\widetilde{\overline{u}}'_{j,n}\widetilde{\overline{\overline{u}}}_{j,n} & 0 \end{bmatrix}, \quad g_{j,n} = \frac{1}{n}\begin{bmatrix} \widetilde{u}'_{j,n}\widetilde{u}_{j,n} \\ \widetilde{\overline{u}}'_{j,n}\widetilde{\overline{u}}_{j,n} \\ \widetilde{u}'_{j,n}\widetilde{\overline{u}}_{j,n} \end{bmatrix}$$

with $\widetilde{\overline{u}}_{j,n} = W_n\widetilde{u}_{j,n}$, and $\widetilde{\overline{\overline{u}}}_{j,n} = W_n^2\widetilde{u}_{j,n}$. An alternative way of estimating the parameters ρ_j and σ_{jj} could be through maximum likelihood estimators. The advantage to using this generalized method of moments estimator is that it is computationally simple and does not depend on normality assumptions like a maximum likelihood estimator would in this situation.

With spatial autoregressive parameters consistently estimated, a Cochrane-Orcutt transformation can then be applied as shown in equation 25. The estimator $\widetilde{\rho}_{j,n}$ can replace ρ_j which allows for δ_k to be estimated using more information:

$$\hat{\delta}^F_{j,n} = \left[\hat{Z}^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)' Z^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)\right]^{-1}\hat{Z}^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)' y^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) \tag{29}$$

with

$$\hat{Z}^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) = P_H Z^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)$$
$$Z^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) = Z_{j,n} - \widetilde{\rho}_{j,n} W_n Z_{j,n}$$
$$y^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) = y_{j,n} - \widetilde{\rho}_{j,n} W_n y_{j,n}$$

The estimators derived from equations 29 and 27 are identical if $\rho_j = 0$. This gives reason to tests for spatial dependence for each equation in the system. If the residuals from each equation in 28 do not exhibit spatial dependence, then the general spatial two-stage least squares (GS2SLS) procedure is not necessary.

Utilizing all relevant information about the correlation across equations, the full information generalized spatial three stage least squares (FGS3SLS) estimator is defined:

$$\breve{\delta}^F_n = \left[\hat{Z}^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)' \left(\hat{\Sigma}^{-1}_n \otimes I_n\right) Z^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)\right]^{-1} \hat{Z}^*_{j,n}\left(\widetilde{\rho}_{j,n}\right)' \left(\hat{\Sigma}^{-1}_n \otimes I_n\right) y^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) \tag{30}$$

where $\hat{\Sigma}_n$ is estimated as a $m \times m$ matrix whose (j,l)-th element is:

$$\hat{\sigma}_{jl,n} = n^{-1} \widetilde{\varepsilon}'_{j,n} \widetilde{\varepsilon}_{l,n}$$
$$\widetilde{\varepsilon}_{j,n} = y^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) - Z^*_{j,n}\left(\widetilde{\rho}_{j,n}\right) \breve{\delta}^F_{j,n}$$

This estimator is asymptotically normal under certain conditions given in Kelejian and Prucha (2004). The FGS3SLS is considered the preferred estimator for both the Population-Employment Model and Extended Model for Broadband as it accounts for feedback, spatial, and cross-equation effects, all of which are theorized to be important components in the models.

4.3 Identification Strategy

The models involving two and three equations in the system – i.e., equations 8 and 16 – provide a starting point for our estimation procedure. In both the Population-Employment and Extended models, the equilibrium adjustment parameter (λ_E, λ_P, and λ_B) is identified in each equation from the lagged dependent variable. This distinction allows for all other parameters to be identified using some algebraic manipulations of the other estimated parameters which are a combination of multiple structural parameters.

4.3.1 Structural Parameter Estimation

Estimation of the system of equations in 8 and 16 gives a reduced form estimate of the parameters of interest. In order to correctly assess causality, the structural parameters need to be recovered and tested as described in tables 1 and 2. The parameters from equation 8 of the Population-Employment Model can be illustrated in vector form:

$$\delta = \begin{bmatrix} A_1 \\ A_2 \\ \Phi_E \\ \frac{A_2}{\lambda_E} \\ \frac{\Phi_E}{\lambda_E} \\ -\lambda_P \\ B_1 \\ B_2 \\ \Phi_P \\ \frac{B_2}{\lambda_P} \\ \frac{\Phi_P}{\lambda_P} \\ -\lambda_E \end{bmatrix} = \begin{bmatrix} \alpha_1\lambda_P \\ \alpha_2\lambda_P \\ \alpha_2\phi_E\lambda_P \\ \frac{\alpha_2\lambda_P}{\lambda_E} \\ \frac{\alpha_2\phi_E\lambda_P}{\lambda_E} \\ -\lambda_P \\ \beta_1\lambda_E \\ \beta_2\lambda_E \\ \beta_2\phi_P\lambda_E \\ \frac{\beta_2\lambda_E}{\lambda_P} \\ \frac{\beta_2\phi_P\lambda_E}{\lambda_P} \\ -\lambda_E \end{bmatrix} \tag{31}$$

An unrestricted estimation procedure would not account for the interconnectedness of the variables with the equilibrium lagged adjustment parameters λ_P and λ_E. In order to test the hypotheses for the Population-Employment Model as shown in table 1, I need to consider the distinction between A_i and α_i as well as B_i and β_i. By applying the delta method, I can recover estimators for the structural parameters (Hayashi, 2000). Consider q nonlinear restrictions:

$$r\left(\delta_0\right) = 0 \tag{32}$$

where $r(\cdot)$ is a q-vector valued function and δ_0 is the true value of the parameters of interest. With a candidate estimator, $\hat{\delta}$, taking a first order Taylor's series expansion about $r\left(\hat{\delta}\right)$ about δ_0 leads to:

$$r\left(\hat{\delta}\right) = r\left(\delta_0\right) + R\left(\delta^*\right) r\left(\hat{\delta} - \delta_0\right) \tag{33}$$

where δ^* is a convex combination of $\hat{\delta}$ and δ_0 and $R\left(\cdot\right)$ is the derivative of $r\left(\cdot\right)$ with respect to the parameter vector δ. With a consistent estimator of $\hat{\delta}$, I can replace δ^* by δ_0 which leads to an asymptotic result:

$$\sqrt{n} r\left(\hat{\delta}\right) \simeq \sqrt{n} R\left(\delta_0\right)\left(\hat{\delta} - \delta_0\right) \tag{34}$$

The above result allows us to constructing a Wald test statistic which handles non-linear restrictions:

$$W \equiv r(\hat{\delta})' \left[R(\hat{\delta})\hat{\Sigma}R(\hat{\delta})'\right]^{-1} r(\hat{\delta}) \underset{d}{\rightarrow} \chi^2(\#r) \tag{35}$$

where $\#r$ is the number of restrictions imposed in the function $r(\hat{\delta})$.

In the Population-Employment Model, there are four parameters of interest: $\alpha_{1,BB}$, α_2, $\beta_{1,BB}$, and β_2. Let $r(\cdot)$ be a function of the parameters from the model, then I can rearrange the estimated reduced form parameters in such a way to identify the structural parameters of interest:

$$r_0\left(\delta\right) = \begin{bmatrix} \frac{A_1}{\lambda_P} \\ \frac{A_2}{\lambda_P} \\ \frac{B_1}{\lambda_E} \\ \frac{B_2}{\lambda_E} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} \tag{36}$$

where I would like to evaluate each of these restrictions one at a time as well as across equations. This can be done by constructing a Wald test at which I assume the true parameter δ

and take a Taylor series expansion about this value. This allows for inference upon the null hypothesis of $H_0 : r_0(\delta) = 0$:[13]

$$R_0(\delta) \equiv \frac{\partial r_0(\delta)}{\partial \delta'} = \begin{bmatrix} \frac{1}{\lambda_P} & 0 & 0 & 0 \\ 0 & \frac{1}{\lambda_P} & 0 & 0 \\ 0 & \frac{A_2}{\lambda_P \Phi_E} & 0 & 0 \\ 0 & \frac{\lambda_E}{\lambda_P} & \frac{B_1}{A_2} & \frac{B_2}{A_2} \\ 0 & \frac{A_2 \lambda_E}{\lambda_P \Phi_E} & \frac{B_1}{\Phi_E} & \frac{B_2}{\Phi_E} \\ \frac{A_1}{\lambda_P^2} & \frac{A_2}{\lambda_P^2} & 0 & 0 \\ 0 & 0 & \frac{1}{\lambda_E} & 0 \\ 0 & 0 & 0 & \frac{1}{\lambda_E} \\ 0 & 0 & 0 & \frac{B_2}{\lambda_E \Phi_P} \\ \frac{A_1}{B_2} & \frac{A_2}{B_2} & 0 & \frac{\lambda_P}{\lambda_E} \\ \frac{A_1}{\Phi_P} & \frac{A_2}{\Phi_P} & 0 & \frac{B_2 \lambda_P}{\lambda_E \Phi_P} \\ 0 & 0 & \frac{B_1}{\lambda_E^2} & \frac{B_2}{\lambda_E^2} \end{bmatrix}' \tag{37}$$

The test can allow for testing of multiple restrictions, which is important in distinguishing whether or not α_2 and β_2 are jointly different from 0. The comparable derivation for the Extended Model for Broadband can be found in the Appendix Section A.2.

4.3.2 Moran's I Tests

Use of spatial econometric methods can help correct for biased estimators when spatial dependence is present in the data. At the same time, excessive modeling of spatial processes when there is no evidence for spatial dependence may lead to inefficient estimators and adds unnecessary complexity to a model. Therefore, testing for spatial dependence is a crucial first step to determine if the more complicated estimators (2SLS and FGS3SLS) are necessary. Moran's I Test for spatial dependence is applied to justify the need for the spatial methods employed in this paper. First proposed in Moran (1950), the Moran's I test is used to detect spatial dependence in an irregular lattice process.[14] The test statistic is defined as:

[13] This involves taking the derivative of equation 36 with respect to the true parameters in δ. Note that the above matrix is transposed so that each row in the matrix reflects the derivative of the first restriction with respect to the parameter vector δ.

[14] Spatial data can be thought of as resulting from observations on the stochastic process:

$$\mathbf{Z}(\mathbf{s}) : \mathbf{s} \in D \subset \mathbf{R}^d$$

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(X_i - \bar{X})(X_j - \bar{X})}{\sum_i (X_i - \bar{X})^2} \tag{38}$$

where N is the number of locations indexed by i and j, X is the variable of interest, $\bar{X}$ is the mean of X, and w_{ij} is an element of the spatial weights matrix which defines the neighbors for the process X. Under the null hypothesis of no spatial dependence, the expectation of the Moran's I test statistic is $\frac{-1}{N-1}$. The value of the test statistic will approach 0 under the null of no spatial dependence as the number of locations increases. The test statistic can take on values between -1 and 1, where a test statistic approaching 1 results in positive dependence (i.e. high values are clustered by high values as well as low values clustered by low values). A test statistic close to -1 indicates negative dependence, which results in repulsion of the variable of interest. In a sense, although not directly, a Moran's I Test Statistic is analogous to a simple correlation coefficient which also ranges from -1 to 1 with the interpretation of negative and positive correlation.

As described in Kelejian and Prucha (2001), the residuals from the two-stage least squares estimators of equation 27 can be tested using the proposed Moran's I Test to determine if a Cochrane-Orcutt Transformation of the data is justified. If the residuals indicate a rejection of the null hypothesis, estimating the spatial autoregressive parameters ρ_j can help correct for the potentially unobservable spatial process that is manifesting itself through spatial dependence in the data.

4.3.3 Structural Spatial Parameters

A further concern for spatial processes involved in the data arise from defining the markets for population, employment, and broadband through a location's neighbors. This was origi-

where Z is a variable observed at a location s defined using some $(d \times 1)$ vector of coordinates. In lattice data, the domain D is fixed and discrete with a countable number of mutually exclusive locations. Where a regular lattice involves an equally spaced grid of locations, this also implies that each location (except on the edges) has the same number of neighbors. With an irregular lattice, locations are not equally spaced on a grid and locations generally do not have the same number of neighbors. Lattice data are also referred to as areal data.

nally proposed in Boarnet (1994) for the residential and labor markets as a way to account for households commuting across municipal boundaries to their place of employment. In the two equation model, I can use the parameter vector from equation 31 to construct a restriction vector to test whether the spatial parameters, ϕ_P and ϕ_E, influence the model. I can construct a Wald Test via the Delta Method in a similar fashion to the structural parameters in section 4.3.1:

$$r_1(\delta) = \begin{bmatrix} \frac{\Phi_E}{A_2} \\ \frac{\Phi_P}{B_2} \end{bmatrix} = \begin{bmatrix} \phi_E \\ \phi_P \end{bmatrix} \tag{39}$$

Taking the derivative of $r_1(\delta)$ with respect to δ' in equation 31 yields:

$$R_1(\delta) \equiv \frac{\partial r_1(\delta)}{\partial \delta'} = \begin{bmatrix} 0 & 0 \\ \Phi_E & 0 \\ \frac{1}{A_2} & 0 \\ \Phi_E \lambda_E & 0 \\ \frac{\lambda_E}{A_2} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \Phi_P \\ 0 & \frac{1}{B_2} \\ 0 & \Phi_P \lambda_P \\ 0 & \frac{\lambda_P}{B_2} \\ 0 & 0 \end{bmatrix}' \tag{40}$$

The comparable derivatives for Extended Model for Broadband can be found in Appendix Section A.2.1

5 Data Description

The units of observation for this study are counties in the contiguous 48 states of the US over the period spanning from 2008 to 2012. Table 3 provides some summary statistics on the endogenous variables of interest

Table 3: Summary Statistics

	2008		2010		Δ 2008 to 2010	
Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
# of BB Providers	12.4	6.9	12.2	6.6	-0.2	2.0
–Neighbors	12.7	5.1	12.5	5.0	-0.2	1.1
Employed	42,312	149,530	40,088	140,464	-2,224	9,763
–Neighbors	44,217	92,396	41,910	86,998	-2,307	5,836
Establishments	2,747	10,783	2,719	10,798	-29	284
–Neighbors	2,843	6,613	2,814	6,626	-29	169
Population	97,161	309,345	98,833	314,772	1,672	7,107
–Neighbors	101,297	202,953	103,071	206,785	1,775	4,646
Migration	—	—	—	—	39	3,494
–Neighbors	—	—	—	—	61	1,800

Sample includes 3,109 counties in the lower 48 states of the United States.

5.1 Broadband Data

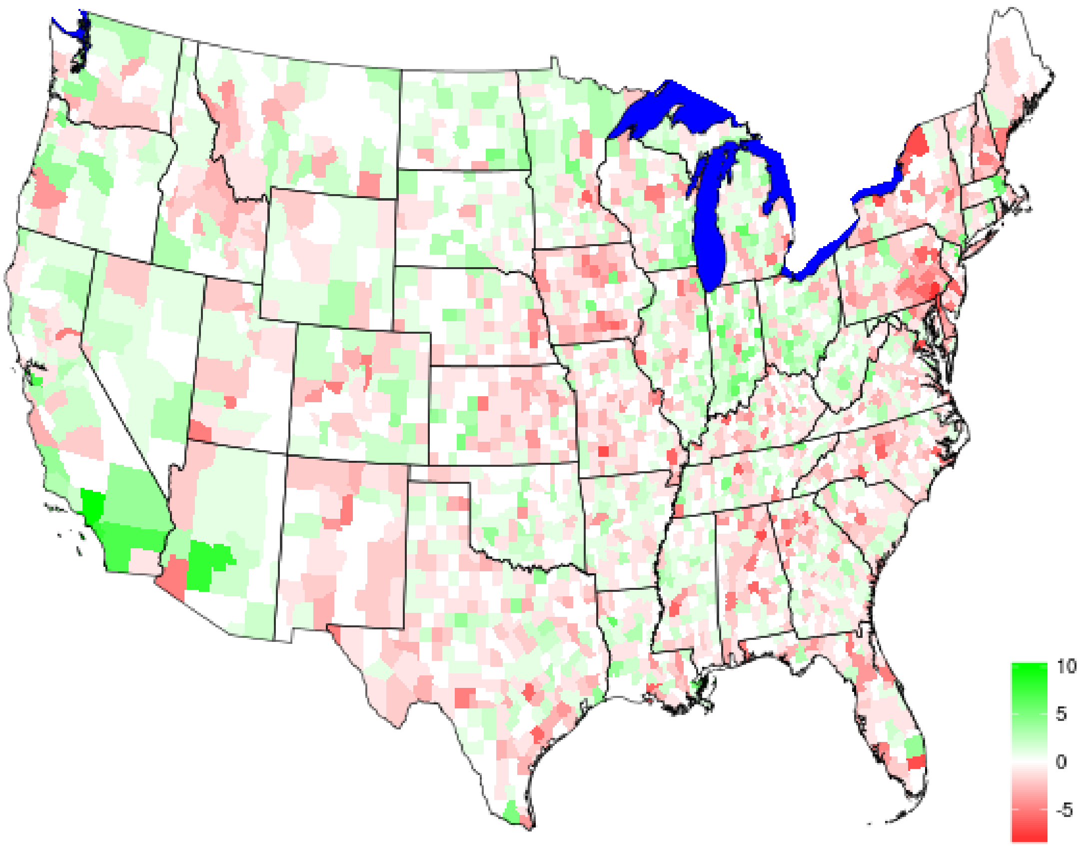
Data on broadband availability were taken from the FCC Form 477 database.[15] For each county, the FCC reports the total number of broadband providers. The number of providers reflects the supply of broadband services within the county which proxies the broadband availability for an area.

There are limitations with these proxies, however. For total number of providers, there is no accounting for the size, quality, or price these firms provide. Neither does the data distinguish between firms providing service to households, to firms, or to both. For confidentiality purposes, counties with 1-3 providers are suppressed so as to not identify the providers.[16] Most importantly, the data obscures consolidation among local broadband providers. The impact of this, in many circumstances, would be fewer providers but no change (or even growth) in the local supply of broadband services. There is no reason to believe that some areas are systematically affected more by consolidation than others, which would create a bias in estimates. However, because broadband services are likely mismeasured, the result

[15] The FCC definition of broadband is providers offering, or subscribers with, fixed-location Internet access connections faster than 200 kilo-bits per second.

[16] In cases where the data are suppressed, observations were recoded to take a value of 2 to be consistent with similar studies in the literature using FCC data.

Figure 1: US Broadband Availability Change from 2008 to 2010



of which is that the uncertainty of the estimators increases and should be reflected in the standard errors.

5.2 Employment Data

County level data on total employment and number of establishments were taken from the Quarterly Census of Employment and Wages (QCEW) of the Bureau of Labor Statistics (BLS). The QCEW serves as a near census of monthly employment and quarterly wages by 6-digit North American Industry Classification System (NAICS) at the county level. Data are suppressed when there are fewer than three firms in an industry, where the employment of a single firm accounts for over 80 percent of an industry, or at the special request of a State if there is reason to believe that disclosure of information pertaining to an industry would violate a State's disclosure provisions.[17]

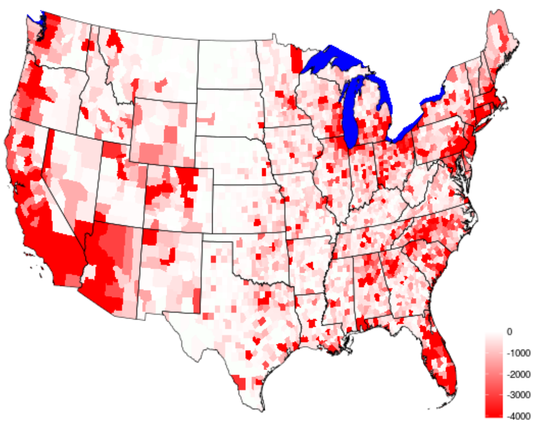


Figure 2: *Employment Change from 2008 to 2010*

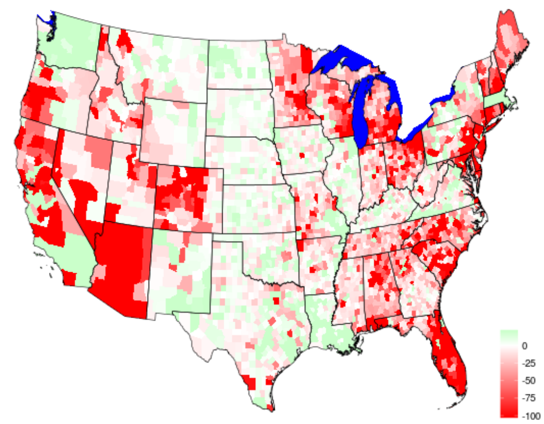


Figure 3: *Establishment Change from 2008 to 2010*

The effects of The Great Recession can be seen to be widespread across the United States, although not necessarily of the same magnitude, with figures 2 and 3. While the change in employment was overwhelmingly negative across the United States, the change in number of establishments was positive in some areas during this time. An establishment is defined by the BLS as "a single economic unit, such as a farm, a mine, a factory, or a store, that produces goods or services." The net change in establishments between 2008 and 2010 does not give insight to total births and total deaths of establishments[18].

5.3 Population Data

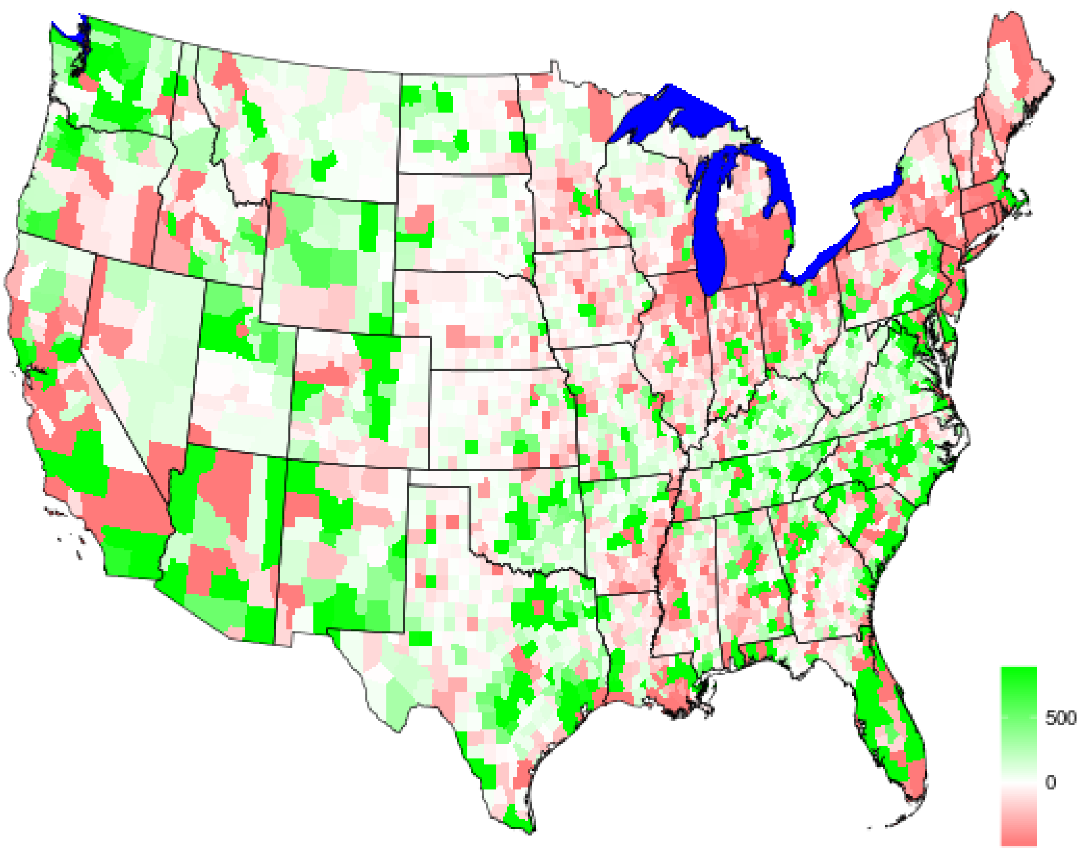
The Census Bureau provides county level estimates of population from 2008 to 2010. Since the Census of Population occurs at the beginning of each decade, county level population

[17] Suppression is found in the data for some NAICS industry classes at the county level, but it is not an issue when all NAICS industry classes are aggregated at the county level.

[18] In Kolko (2012), he uses the National Establishment Time Series (NETS) data which is a panel of all establishments in the United States. These data can give insight into the total births and deaths of establishments. However, due to financial constraints, this paper is unable to use such data.

for 2010 was taken from the 2010 Census of Population and Housing. For 2008 and 2010, the Intercensal Population and Housing Unit Estimates provided by the Census were used to obtain population estimates. These estimates of county level population are based on the expected birth rates and death rates from the initial demographics of the county. The 2008 estimate is bench-marked to the 2000 Census while the 2010 estimate is bench-marked to the 2010 Census.

Figure 4: Net Migration from 2008 to 2010



The changes in population levels for each county are estimated using demography models and therefore it is not appropriate to think of population changes between 2008 to 2010 to be movements in populations or households. Instead, migration data from the Internal

Revenue Services (IRS) is used for changes in population. These data come from filed tax returns using Form 1040, 1040A, and 1040EZ processed by the IRS for all returns filed by late September of the filing year. This covers 95 to 98 percent of the individual income tax filing population with poor and rich people underrepresented in this data because they are groups most likely to not file taxes or file taxes after September. A migrant is determined by the Census checking codes in tax forms with the previous year's tax forms and comparing geographic codes. Individuals are presumed to migrate when the geographic codes do not match. From these data provided (by the Census), the IRS can then determine the previous county of residence and their destination county of residence for said tax year. These data are then aggregated for each origin-destination county pair to provide number of returns (approximates households), number of exemptions (approximates population), and aggregate adjusted gross income for each origin to destination.

The pattern of migration across the United States during this time can be seen in figure 4. The migration patterns for this two year window is similar to the migration patterns that have been seen within the United States since 2000, although one side-effect of The Great Recession was that the migration rates slowed during this two year window.

5.4 Control Variables

Within both the Population-Employment and Extended models, controls are chose to conform to those found in the literature. The set of control variables in the **Z** vectors serve as initial values which determine the equilibrium levels that a regional economy will converge toward in the population, employment, and broadband equations. I choose to use 2008 as the year for the initial conditions of control variables so that they will still be related to a change across the years of 2008 to 2010 but also are not simultaneously determined by changes in population or employment as the values in 2010 would be. Lagging the variables too much would run the risk of being uncorrelated with any current variables in the system and fail to identify any parameters of interest. The control variables are chosen to be consistent with the literature on broadband, migration, and firm location studies.

Table 4: Summary Statistics: Control Variables

Variable	2008 Mean	Standard Deviation
Median Home Value	\$130,914	79,509
Median Household Income	\$45,463	11,778
Average Wages (weekly)	\$633	142
Taxable Wages (annual)	\$9,000	3,531
Unemployment Rate	5.76%	2.07
Housing Permit Start-ups	446	1,549
Share with Bachelor's Degree	19.4%	8.76
Natural Amenity Index	.0559	2.28
Percent Vacation Home Share	2.96%	.232
TPI	.0043	.499
Highway Density ($\times 10^{-4}$)	2.06%	1.41

Sample includes 3,109 counties in the lower 48 states of the United States.

County unemployment rates were obtained from the Local Area Unemployment Statistics (LAUS) from the BLS. This monthly series tracks total employed and unemployed at the county level and provides an estimate for the annual unemployment rate that is seasonally adjusted. This is a control for labor market conditions and is used only in the employment equation.

The Census Bureau also provides county level data on percent of population with a bachelor's degree or higher, median value of owner-occupied housing units, median household income, and share of vacation homes. Percent of population with a bachelor's degree or higher is used in the employment equation to control for firms locating in areas with differing levels of human capital. Median value of owner-occupied housing units and median household income are used as controls in the population equation as a way to proxy for cost of living as well as the types of households that reside in the county. Share of vacation homes are a control in the population equation to account for any retirees, those presumably unaffected by labor market conditions but still mobile, within a county. Data on permits granted to housing start-ups at the county level is also used as an instrument for broadband providers.

Areas with older homes face higher costs for providing broadband access to than newer homes. Because of the shared fixed costs of infrastructure with newer housing developments, broadband providers should be positively correlated with housing permits.

Additional non-economic controls include the density of roads in a county, a natural amenity index for each county, and topography index for each county. Road density was calculated with a 2005 ESRI shape file of roads and ArcGIS software. Road density is the percentage of land area that highway covers in a county and is used in the employment equation to proxy potential transportation costs that a firm may encounter. The USDA-ERS provides data for the natural amenity index. The index takes into account temperate temperatures, hours of sunlight, relative humidity, and the spatial variation of land characteristics within counties to approximate scenic views (e.g., plains, tablelands, plains with hills or mountains, open hills or mountains, and hills and mountains). The natural amenity index is highly correlated with population movements since the 1970s in the United States and is used in the population equation McGranahan (1999).

A topography index variable is calculated through shapefiles from `DIVA-GIS.org` and the R function `terrain()` from the raster package. The index is the topographic position index (TPI) which is used to calculate topographic slope positions within a grid where the higher values are associated with more rugged terrain[19]. This is used as an instrument for broadband providers in all equations as Kolko (2012) demonstrated that costs to deploy broadband are correlated with the terrain characteristics.

6 Results

This section presents results for both the Population-Employment Model and Extended Model for Broadband as well as a discussion on the implications of the findings. Results are presented for the Population-Employment Model and Extended Model for Broadband, and

[19] Similar topological indexes were considered (such as Terrain Ruggedness Index) and gave qualitatively similar results.

then robustness checks across NAICS industry classifications are performed.

As an initial diagnostic test to assess the applicability of spatial methods, the Moran's I tests for spatial dependence of total employment (0.24), total establishments (0.23), population (0.13), and broadband providers (0.53), all strongly reject the null hypothesis of spatial independence (all p-values <0.001). The spatial weights matrix for the test are based on a contiguity measure of neighbors. Other specifications for neighbors were also used,[20] these similarly resulted in rejection of the null hypothesis of spatial independence. Although these variables separately indicate spatial dependence, this does not necessarily imply that a model of employment growth with lagged population and broadband growth will also exhibit spatial dependence but they do merit further spatial analysis.

6.1 Population-Employment Model

6.1.1 Reduced Form Results

The Population-Employment Model is estimated with three different estimators: Ordinary Least Squares (OLS), Generalized Spatial Two-Stage Least Squares (GS2SLS), and Full Information Generalized Three-Stage Least Squares (FGS3SLS).[21] There are two different specifications for employment in order to try and tease out differing aspects of the labor market: total employed and number of establishments. Results are presented in tables 5 and 6 for the reduced form parameters with each column indicating a different estimator.

With the OLS results, there is no correction for the simultaneous relationship between population and employment or any spatial effects. The OLS results can be seen as a naïve approach to evaluating the Population-Employment Model. The GS2SLS estimator, based on equation 29, corrects for simultaneity and spatial dependence for the labor market effects associated with population and employment. The FGS3SLS estimator comes from

[20] Distance based measure and k-nearest neighbors were used with varying levels of distance and k.

[21] For computational purposes, all of the variables have been transformed to z-scores for efficient inversion of matrices in the estimation procedures. Since the purpose of estimation is inference on whether or not structural parameters differ statistically from zero, this does not alter the interpretation of Wald tests.

Table 5: Population-Employment Model: Reduced Form Population Equation (ΔP)

Emp. Variable:	Total Employment			No. of Establishments		
Covariates	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
BB_0 ($A_{1,BB}$)	0.2907°	0.5381•	0.3939°	0.2689•	0.3642•	0.2774
	(0.1406)	(0.1709)	(0.1719)	(0.1022)	(0.1289)	(0.2777)
E_{t-1} (A_2)	0.3902	0.5725	1.1823	-0.9084	-0.6801†	-0.3439
	(0.7306)	(0.6535)	(0.741)	(0.7672)	(0.3903)	(0.7369)
$W_E E_{t-1}$ (Φ_E)	-0.4074	-1.0398†	-0.8902	0.0745	0.1095	0.2129
	(0.2807)	(0.619)	(0.8318)	(0.1251)	(0.094)	(0.1775)
ΔE ($\frac{A_2}{\lambda_E}$)	0.7126†	1.0959	1.937†	0.0541	-0.4571	-1.3787†
	(0.3868)	(0.9657)	(1.1258)	(0.0873)	(0.3224)	(0.8117)
$W_E \Delta E$ ($\frac{\Phi_E}{\lambda_E}$)	-0.5918°	-1.2457†	-1.1183	-0.014	0.2663	0.5367
	(0.2983)	(0.6931)	(0.9393)	(0.0933)	(0.2942)	(0.5283)
P_{t-1} ($-\lambda_P$)	-0.4708	-0.3889	-0.1235	0.1459	-0.1214	-0.4275
	(0.7263)	(0.5321)	(0.6676)	(0.599)	(0.4695)	(0.9831)
Moran's I statistic	0.1647	0.0179	-0.0393	0.1494	0.0148	-0.1281
P-Value	0	0.0863	0.0002	0	0.154	0

Robust standard errors in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by •, °, and †, respectively. Moran's I test is on the residuals with a spatial weight matrix based on contiguity of counties. Controls include: median home value, natural amenity index, share of vacation homes, share of population above 65, share of population between 4-17 in poverty, and a dummy for rural counties.

equation 30 and further extends the GS2SLS estimator by accounting for cross-equation correlation in the system of equations. The FGS3SLS is preferred to the GS2SLS estimator; both are preferred to the OLS estimator.

The parameters of interest are the coefficients associated with the broadband variable, BB_0, in the population equation ($A_{1,BB}$) and the employment equation ($B_{1,BB}$). Inspection of these reduced form parameters indicates that there may be some evidence that "people follow broadband" via inspection of all of the estimators except for FGS3SLS, where number of establishments is used for the employment variable. The positive, and significant, relationship between total number of broadband providers in 2008 and change in county population from 2008 to 2010 indicates that more broadband providers is associated with an increase

Table 6: Population-Employment Model: Reduced Form Employment Equation (ΔE)

Emp. Variable:	Total Employment			No. of Establishments		
Covariates	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
BB_0 ($B_{1,BB}$)	0.0114	0.0988†	0.0619	-0.3121$^\bullet$	-0.1991	0.1174
	(0.0229)	(0.0531)	(0.0523)	(0.0843)	(0.1675)	(0.2452)
P_{t-1} (B_2)	-0.2187	-0.2806†	-0.2301	-0.125	-0.1706	-0.221
	(0.2584)	(0.164)	(0.176)	(0.8529)	(0.6474)	(0.6794)
$W_P P_{t-1}$ (Φ_P)	-0.0911	-0.0632	-0.0625	0.0659	0.1182	0.0909
	(0.076)	(0.048)	(0.0509)	(0.1568)	(0.1285)	(0.1897)
ΔP ($\frac{B_2}{\lambda_P}$)	0.1115°	0.033	0.1096	0.0836	-0.1104	-0.6634
	(0.0511)	(0.1248)	(0.1185)	(0.1291)	(0.3942)	(0.6041)
$W_P \Delta P$ ($\frac{\Phi_P}{\lambda_P}$)	-0.0707†	-0.0131	0.0007	-0.131	-0.0472	-0.1649
	(0.0429)	(0.0574)	(0.0566)	(0.0962)	(0.1202)	(0.2956)
E_{t-1} ($-\lambda_E$)	-0.7272$^\bullet$	-0.731$^\bullet$	-0.7136$^\bullet$	0.2733	0.1567	-0.2556
	(0.2615)	(0.1294)	(0.1422)	(0.7489)	(0.4416)	(0.5616)
Moran's I statistic	0.2062	0.0227	0.0168	0.208	-0.0019	-0.0085
P-Value	0	0.0303	0.1073	0	0.8789	0.4443

Robust standard errors in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by $^\bullet$, $^\circ$, and †, respectively. Moran's I test is on the residuals with a spatial weight matrix based on contiguity of counties. Controls include: percentage of highways in county, share of population with at least a bachelor's degree, average weekly wage, share of taxable wages, unemployment rate, median household income, share of population above 65, and a dummy for rural counties.

in residents for a county. Because all variables in the regression have been transformed to z-scores, the interpretation of the coefficients are in terms of standard deviation increases in variables. This necessitates the use of the data presented in table 3 in order to address the magnitude of effects. For example, the FGS3SLS estimator with total employment as the employment variable indicates that a one standard deviation increase in total number of broadband service providers in 2008 (approximately 7) would lead to an increase of 0.3939 standard deviations of migrants (one standard deviation is approximately 3,494 people). So the effects of a county having one more broadband service provider would be, on average, associated with 200 more migrants between the period of 2008 to 2010.

The coefficients on BB_0 in the employment equation (table 6) are not significantly different from zero. Thus, no significant relationship between broadband and either total employment or number of establishments is evident. In other words, the data do not support the claim that broadband is a driver of job growth.

Another relationship of interest for this model is to compare the endogenous variables, ΔP and ΔE, as well as the lagged values of employment and population to determine if "jobs follow people" or "people follow jobs." For the Population equation, the results are mixed across estimators as well as the definition of the employment variable. With the FGS3SLS estimator for the population equation, there appears to be evidence that "people follow jobs" when employment is defined as total employed. However, this result reverses to "jobs repel people" if the definition of employment changes to number of establishments. While these two interpretations appear to be at odds with each other, one explanation could be that consolidation of firms within an industry could be occurring at this time. A limitation of the data available is that this explanation cannot be tested without knowing the composition of firms within a county[22].

6.1.2 Structural Parameter Tests

Because reduced form parameters are a function of the structural parameters of interest, I need to recover the structural parameters and test whether they are statistically different from zero, a necessary condition for causality. This is done through a Wald test described in equation 35 and constructed for each estimator. Whether or not "jobs follow broadband" or "people follow broadband" are the primary interest here. The preferred estimator to evaluate this claim is the FGS3SLS estimator as it accounts for spatial autocorrelation, simultaneity, and cross equation correlation.

Whereas in the reduced form estimates one may have concluded that "people follow broadband," the structural parameter of $\alpha_{1,BB}$ does not significantly differ from zero in

[22] More finely scaled data, such as NETS data, may be able to address this result. Because the focus of this paper is on broadband and not necessarily the population and employment relationship, this is left for future research.

Table 7: Population-Employment Model: Wald Tests for Structural Parameters

Emp. Variable:	Total Employment			No. of Establishments		
	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
Null Hypothesis						
$\alpha_{1,BB} = 0$	0.4909	0.6116	0.0377	0.0615	0.0704	0.1517
	(0.2813)	(0.2478)	(0.6077)	(0.5575)	(0.5426)	(0.4495)
$\beta_{1,BB} = 0$	0.1984	0.329	0.4245	0.1176	0.0991	0.1316
	(0.4135)	(0.3414)	(0.3034)	(0.4819)	(0.503)	(0.4678)
$\alpha_1 = \beta_1 = 0$	0.711	0.6137	0.5183	0.1869	0.1608	0.2981
	(0.7008)	(0.7358)	(0.7717)	(0.9108)	(0.9227)	(0.8615)
$\alpha_2 = 0$	0.1899	0.0548	0.0169	0.0705	0.0477	0.0499
	(0.4195)	(0.5698)	(0.6785)	(0.5423)	(0.5841)	(0.5795)
$\beta_2 = 0$	0.318	0.3032	0.3876	0.0232	0.0683	0.0297
	(0.3464)	(0.3534)	(0.3171)	(0.652)	(0.5459)	(0.63)
$\alpha_2 = \beta_2 = 0$	0.3365	0.7522	0.6212	0.0932	0.1052	0.0736
	(0.8451)	(0.6865)	(0.733)	(0.9545)	(0.9487)	(0.9639)
$\phi_P = 0$	0.6223	0.3372	0.0951	0.2715	0.7714	1.0912
	(0.2451)	(0.3378)	(0.5079)	(0.3694)	(0.2125)	(0.1611)
$\phi_E = 0$	1.4175	2.0678*	1.6966	0.1596	0.7474	0.1073
	(0.1247)	(0.0783)	(0.1015)	(0.4428)	(0.2173)	(0.4933)

P-Values in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by $^{\bullet}$, $^{\circ}$, *and* †, *respectively.*

any of the Wald tests constructed. The same can be said about the structural relationship between broadband and employment as seen for the Wald tests for $\beta_{1,BB}$. Given that a Wald test typically over-rejects the null hypothesis compared to similar tests of non-linear restrictions such as a Lagrange multiplier or likelihood ratio tests (Hayashi, 2000), the lack of a significant relationship for both of these parameters casts doubt on claims that broadband access can be a driver of economic growth for a region. This result is driven by accounting for movements *towards* equilibrium levels in population and employment, which are the λ parameters in each equation. These parameters, λ_P and λ_E, are identified from the lagged level of population or employment, respectively, and represent the rate at which counties are converging toward an equilibrium level. In order for the system to be stable, these parameters need to be between 0 and 1 (details of which are described in Appendix A.1). Given that these parameters indicate a stable system, they help in identifying how broadband affects population and employment movements, and within this system the effects of broadband are not significant.

A secondary concern is whether or not "jobs follow people" or "people follow jobs" in the system. Identifying this relationship also relies on recovering the structural parameters in a similar manner to evaluating if broadband impacts population or employment. While the reduced form parameters are mixed for interpreting this relationship, the structural parameters fail to indicate a significant relationship between the two. Further, the spatial relationship between the housing and labor markets, ϕ_E and ϕ_P respectively, are recovered and tested via a Wald test. However, the housing and labor market effects do not appear to be significantly different from zero. This is to be expected given that the relationship between population and employment is also not significant. This gives concern that the model may not contain housing and labor markets defined spatially; however, if the models are estimated with the spatial housing and labor markets removed, the main results remain. I am unable to detect a significant, structural relationship between broadband and population and employment.

6.2 Extended Model for Broadband

6.2.1 Reduced Form Results

The Extended Model for Broadband is a richer model that accounts for potential endogeneity of the broadband deployment decision at the county level. Like the Population-Employment Model, this model is estimated via OLS, GS2SLS, and FGS3SLS and results are presented as a way assess whether simultaneity is a concern within the system as a whole. As before, two different proxies for employment are used – total employment and number of establishments – in order to gain traction as to whether or not broadband is affecting the productivity of a firm (total employment) as well as generation of new firms (number of establishments) that are potentially utilizing broadband. The Population equation is presented in table 8, the

Employment equation is presented in table 9, and the Broadband equation is presented in table 10.

Table 8: Extended Model: Reduced Form Population Equation (ΔP)

Emp. Variable:	Total Employment			No. of Establishments		
Covariates	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
E_{it-1} (A_2)	0.3245	0.4151	0.5963	-0.9098	-0.8335°	-0.5004
	(0.7231)	(0.5291)	(0.5844)	(0.7041)	(0.3474)	(0.6756)
$\mathbf{W}_E E_{it-1}$ (Φ_{EA})	-0.1929	-0.3208	-0.3877	0.2231	0.5263$^{\bullet}$	0.4217
	(0.333)	(0.64)	(0.7798)	(0.1499)	(0.1372)	(0.261)
ΔE_{it} ($\frac{A_2}{\lambda_E}$)	0.6734†	1.0122	1.3441†	0.0584	-0.3538	-1.3172†
	(0.3819)	(0.6774)	(0.8016)	(0.0851)	(0.2737)	(0.7961)
$\mathbf{W}_E \Delta E_{it}$ ($\frac{\Phi_{EA}}{\lambda_E}$)	-0.5513†	-1.1418†	-1.0572	-0.0569	0.0042	0.5685
	(0.3062)	(0.6797)	(0.8148)	(0.0918)	(0.3028)	(0.6162)
BB_{it-1} (A_3)	0.4455$^{\bullet}$	0.9216$^{\bullet}$	0.9416$^{\bullet}$	0.4145$^{\bullet}$	0.7165$^{\bullet}$	0.7058$^{\bullet}$
	(0.1698)	(0.2123)	(0.1997)	(0.1268)	(0.1774)	(0.2686)
$\mathbf{W}_B BB_{it-1}$ (Φ_{BA})	-0.3325$^{\bullet}$	-0.9794$^{\bullet}$	-0.8345$^{\bullet}$	-0.3113$^{\bullet}$	-0.7319$^{\bullet}$	-0.5018†
	(0.103)	(0.2036)	(0.1935)	(0.0839)	(0.1625)	(0.2953)
ΔBB_{it} ($\frac{A_3}{\lambda_B}$)	0.0864°	0.7797$^{\bullet}$	0.8722$^{\bullet}$	0.0847°	0.7154°	1.0873†
	(0.0382)	(0.2517)	(0.222)	(0.0399)	(0.2893)	(0.5627)
$\mathbf{W}_B \Delta BB_{it}$ ($\frac{\Phi_{BA}}{\lambda_B}$)	-0.0925†	-1.295$^{\bullet}$	-1.0421$^{\bullet}$	-0.0716	-0.8645$^{\bullet}$	-0.7787
	(0.0537)	(0.2761)	(0.2412)	(0.053)	(0.301)	(0.5962)
P_{t-1} ($-\lambda_P$)	-0.5019	-0.4667	-0.3483	0.0977	-0.1064	-0.4858
	(0.7142)	(0.4053)	(0.4906)	(0.5699)	(0.4157)	(0.8741)
Moran's I statistic	0.1552	-0.0142	-0.0966	0.1386	-0.0108	-0.1566
P-Value	0	0.1926	0	0	0.3253	0

Robust standard errors in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by $^{\bullet}$, $^{\circ}$, and †, respectively. Moran's I test is on the residuals with a spatial weight matrix based on contiguity of counties. Controls include: median home value, natural amenity index, share of vacation homes, share of population above 65, share of population between 4-17 in poverty, and a dummy for rural counties.

The coefficients with respect to the lagged broadband variable, BB_{t-1}, in the Population and Employment equations are roughly the equivalent to the two-equation model if the regressions are interpreted to have no structural model behind them. While the two-equation model gives some evidence that "people follow broadband" when the employment variable is total employment, the extended (three-equation) model displays stronger evidence for this

Table 9: Extended Model: Reduced Form Employment Equation (ΔE)

Emp. Variable:	Total Employment			No. of Establishments		
Covariates	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
P_{it-1} (B_2)	-0.2058	-0.2629	-0.2515	-0.0934	-0.1873	-0.2669
	(0.257)	(0.1605)	(0.1687)	(0.8532)	(0.6423)	(0.6615)
$\mathbf{W}_P P_{it-1}$ (Φ_{PB})	-0.1355	-0.1193	-0.1097	0.0107	0.1675	0.1866
	(0.093)	(0.0998)	(0.0965)	(0.1794)	(0.2563)	(0.311)
ΔP_{it} ($\frac{B_2}{\lambda_P}$)	0.1194°	0.0505	0.0816	0.0881	-0.1401	-0.5808
	(0.0526)	(0.1272)	(0.1257)	(0.1289)	(0.4041)	(0.5525)
$\mathbf{W}_P \Delta P_{it}$ ($\frac{\Phi_{PB}}{\lambda_P}$)	-0.0813†	-0.0106	0.0138	-0.1305	0.0322	-0.1279
	(0.0445)	(0.0537)	(0.0603)	(0.0987)	(0.1307)	(0.2884)
BB_{it-1} (B_3)	-0.0164	0.0675	0.0962†	-0.327$^{\bullet}$	-0.0826	0.3434
	(0.0307)	(0.0572)	(0.0527)	(0.1049)	(0.1737)	(0.2446)
$\mathbf{W}_B BB_{it-1}$ (Φ_{BB})	0.0742†	0.0671	0.0554	0.0961	-0.0515	-0.2546
	(0.0435)	(0.0839)	(0.0817)	(0.0849)	(0.1819)	(0.2607)
ΔBB_{it} ($\frac{B_3}{\lambda_B}$)	-0.0073	0.0592	0.1145°	0.0133	0.2564†	0.6802$^{\bullet}$
	(0.0123)	(0.0433)	(0.0459)	(0.0534)	(0.1344)	(0.1831)
$\mathbf{W}_B \Delta BB_{it}$ ($\frac{\Phi_{BB}}{\lambda_B}$)	0.0177	0.099	0.048	0.0663	0.0168	-0.2485
	(0.0198)	(0.1292)	(0.1286)	(0.0601)	(0.2818)	(0.3727)
E_{t-1} ($-\lambda_E$)	-0.7234$^{\bullet}$	-0.7383$^{\bullet}$	-0.7418$^{\bullet}$	0.246	0.0823	-0.288
	(0.2575)	(0.1228)	(0.1347)	(0.7627)	(0.4179)	(0.4781)
Moran's I statistic	0.2062	0.021	0.001	0.2118	0.0034	-0.0502
P-Value	0	0.0454	0.9029	0	0.7294	0

Robust standard errors in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by $^{\bullet}$, $^{\circ}$, *and* †, *respectively. Moran's I test is on the residuals with a spatial weight matrix based on contiguity of counties. Controls include: percentage of highways in county, share of population with at least a bachelor's degree, average weekly wage, share of taxable wages, unemployment rate, median household income, share of population above 65, and a dummy for rural counties.*

claim (previous coefficient estimate of 0.3939 and significant at the 5% level; comparable estimates for the extended model are 0.9416 and significant at the 1% level). Whereas the earlier model implied effects of a county having one more broadband service provider in 2008 is approximately 200 more migrants for a county between 2008 and 2010, the extended model indicates an additional broadband service provider would lead to approximately 470 more migrants. In addition, the coefficient associated with lagged broadband and number

Table 10: Extended Model: Reduced Form Broadband Equation (ΔBB)

Emp. Variable:	Total Employment			No. of Establishments		
Covariates	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
P_{it-1} (Γ_2)	0.1055	-0.2142	-0.0005	0.2092	-0.1397	0.1931
	(0.1197)	(0.3024)	(0.4239)	(0.2028)	(0.426)	(0.6503)
$\mathbf{W}_P P_{it-1}$ ($\Phi_{P\Gamma}$)	-0.0486	-0.0825	0.2891	0.1807	0.1159	0.2192
	(0.2401)	(0.6379)	(1.0615)	(0.2077)	(0.5564)	(0.9566)
ΔP_{it} ($\frac{\Gamma_2}{\lambda_P}$)	0.0776†	-0.4616°	-0.4579	0.0579	-0.4191°	0.1537
	(0.0418)	(0.2099)	(0.3192)	(0.0518)	(0.2096)	(0.3308)
$\mathbf{W}_P \Delta P_{it}$ ($\frac{\Phi_{P\Gamma}}{\lambda_P}$)	-0.0729	-0.0687	0.0238	-0.0691	-0.3026	-0.1402
	(0.0602)	(0.1172)	(0.1484)	(0.0619)	(0.2027)	(0.3313)
E_{it-1} (Γ_3)	0.2391°	0.4979	1.0899$^{\bullet}$	0.0954	-0.167	0.0148
	(0.1139)	(0.34)	(0.4116)	(0.2896)	(0.3088)	(0.5163)
$\mathbf{W}_E E_{it-1}$ ($\Phi_{E\Gamma}$)	0.2221	-0.5498	-0.7449	-0.0424	0.0399	-0.1886
	(0.2715)	(0.6511)	(1.1901)	(0.224)	(0.6)	(0.9971)
ΔE_{it} ($\frac{\Gamma_3}{\lambda_E}$)	-0.034	0.6406	1.5219°	0.0244	0.3146	0.913$^{\bullet}$
	(0.1096)	(0.5831)	(0.7447)	(0.0646)	(0.2803)	(0.27)
$\mathbf{W}_E \Delta E_{it}$ ($\frac{\Phi_{E\Gamma}}{\lambda_E}$)	-0.0018	-0.9888	-0.9148	-0.0204	-0.6155$^{\bullet}$	-0.5171
	(0.1714)	(0.6663)	(0.6348)	(0.0542)	(0.2211)	(0.3352)
BB_{t-1} ($-\lambda_B$)	-0.5926$^{\bullet}$	-0.5989$^{\bullet}$	-0.675$^{\bullet}$	-0.5758$^{\bullet}$	-0.5209$^{\bullet}$	-0.5086$^{\bullet}$
	(0.0341)	(0.0525)	(0.0732)	(0.0429)	(0.0749)	(0.1119)
Moran's I statistic	0.1921	-0.0045	-0.0484	0.1937	0.0021	-0.1564
P-Value	0	0.6927	0	0	0.8179	0

Robust standard errors in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by $^{\bullet}$, $^{\circ}$, and †, respectively. Moran's I test is on the residuals with a spatial weight matrix based on contiguity of counties. Controls include: TPI, median household income, share of vacation homes, percentage of highways in county, average weekly wage, housing permit start-ups, and a dummy for rural counties.

of establishments as the employment variable indicates evidence in favor of "people follow broadband," although the magnitude is smaller. The analysis can be further examined from a non-structural context through the change in broadband providers, ΔBB_{it}, which is not present in the simpler model. For the Population equation, I find evidence in favor of the hypothesis that "people follow broadband" due to significant estimates on the change in broadband providers from 2008 to 2010. This is found whether the employment variable is total employed or number of establishments. The coefficients are normalized, as in the simpler model, and thus table 3 needs to be used to convert coefficients to more interpretable effects. Doing so indicates that an increase of 1 broadband service provider from 2008 to 2010 would approximately result in 1,525 to 1,900 more migrants over this period, depending on if employment is measured in total employment or number of establishments, respectively.

The simpler model does not provide evidence in favor of the "jobs follow broadband" hypothesis; however, in the Employment equation for the Extended Model (table 9) there appears to be some evidence for this claim. The lagged broadband variable is a significant predictor of employment change from 2008 to 2010 when measured as total employment, but not if employment is measured in number of establishments. In practical terms, a one-provider increase in 2008 leads to a change in approximately 135 more employees (jobs) for a county over 2008 to 2010. There is stronger evidence present for the "jobs follow broadband" hypothesis when inspecting the change in broadband providers from 2008 to 2010, which indicates a positive and significant relationship regardless of the variable used for employment. The coefficient estimates would imply that a one-provider increase over the period of 2008 to 2010 leads to approximately 560 more employees or 97 more establishments from 2008 to 2010.

An advantage to estimating the Extended Model for Broadband is that broadband is treated as endogenous and so the claims of whether or not "broadband follows people" or "broadband follows jobs" can be addressed if causality is of concern. In the broadband equation (table 10), none of the coefficients associated with the population variables are significant with the exception of the GS2SLS estimates for change in population. In other words, there is no evidence in favor of the hypothesis that "broadband follows people." Combined with the findings that "people follow broadband" in the Population equation, this result is highly suggestive that broadband growth over 2008 to 2010 (and broadband level in 2008) causes/attracts migrants over the same time period.

When evaluating the effects of employment on the change in broadband providers, lagged employment is significant when the employment variable is total employees but not when the variable is number of establishments, and change in employment is significant for both definitions of employment. This result suggest that “broadband follows jobs” while “jobs follow broadband” also has evidence in favor of it. Combining these results suggests that the relationship between job growth and broadband growth is symbiotic as they both depend upon each other.

Of secondary concern is the relationship between population and employment. For the Employment equation in table 9, the lack of a significant relationship with the population variables does not give any support to the hypothesis that “jobs follow people” as is the case in the simpler model. The Population equation results in table 8 are likewise comparable to what was found using the simpler model; i.e., that “people follow jobs” when the employment variable is total employment but that “jobs repel people” if the employment variable is number of establishments. As with the simpler model, data limitations do not allow further inspection of this perplexing relationship.

6.2.2 Structural Parameters

As noted earlier in discussing the simpler Population-Employment Model, the reduced form estimates do not convey a complete picture of the relationship between population, employment, and broadband because of the equilibrium adjustment parameters. So long as frictions exist in the economy that result in counties converging *toward* equilibrium but not necessarily being in an equilibrium, the structural parameters need to be recovered for evaluation of the relationship among the three variables of interest.

Table 11 presents ten Wald tests to give a complete picture of the dynamics among the three processes. The first nine tests directly result from the necessary conditions for causality in table 2. Within the context of whether or not any of the three endogenous processes “follow” the other, the results do not indicate evidence for a causal relationship as the null hypothesis of no relationship cannot be rejected for the structural parameters. Not finding evidence for a causal relationship is at odds for some of the reduced form results. This

Table 11: Extended Model: Wald Tests for Structural Parameters

Emp. Variable:	Total Employment			No. of Establishments		
	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
Null Hypothesis						
$\alpha_2 = 0$	0.065	0.0276	0	0.0358	0.0542	0.0689
	(0.5514)	(0.6367)	(0.9418)	(0.6125)	(0.5709)	(0.545)
$\alpha_3 = 0$	0.4633	0.549	0	0.0294	0.0612	0.0856
	(0.2901)	(0.2643)	(0.9418)	(0.6311)	(0.558)	(0.5202)
$\alpha_2 = \alpha_3 = 0$	0.5362	0.5606	0	0.0569	0.0642	0.0857
	(0.7648)	(0.7556)	(1)	(0.9719)	(0.9684)	(0.9581)
$\beta_2 = 0$	0.1907	0.0974	0.135	0.002	0.0399	0.0184
	(0.4189)	(0.505)	(0.4645)	(0.8101)	(0.602)	(0.6714)
$\beta_3 = 0$	0.0979	0.076	0.0777	0.0862	0.0158	0.0265
	(0.5044)	(0.5339)	(0.5315)	(0.5195)	(0.6836)	(0.6404)
$\beta_2 = \beta_3 = 0$	0.2016	0.0975	0.1369	0.2322	0.0427	0.0275
	(0.9041)	(0.9524)	(0.9338)	(0.8904)	(0.9789)	(0.9863)
$\gamma_2 = 0$	0.0398	0.0138	0	0.4503	0.0301	0.0248
	(0.6023)	(0.6943)	(0.9706)	(0.2944)	(0.629)	(0.6464)
$\gamma_3 = 0$	0.3949	0.096	0.1248	0.0239	0.0048	0.001
	(0.3143)	(0.5067)	(0.4745)	(0.6494)	(0.765)	(0.8424)
$\gamma_2 = \gamma_3 = 0$	0.4333	0.1152	0.13	0.4631	0.0575	0.0489
	(0.8052)	(0.944)	(0.9371)	(0.7933)	(0.9717)	(0.9758)
BB Equation 49	$19.0937^{\bullet}$	$28.6462^{\bullet}$	$22.2227^{\bullet}$	$21.5648^{\bullet}$	$12.6845^{\dagger}$	3.0665
	(0.0079)	(2e-04)	(0.0023)	(0.003)	(0.0802)	(0.8788)

P-Values in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by $^{\bullet}$, $^{\circ}$, *and* †, *respectively.*

in turn calls into question how one might interpret a reduced form regression in determining a causal relationship for broadband. If a spatial economy has regions that converge towards an equilibrium at some adjustment rate, then this necessitates a structural model to recover the true parameters of interest. The results presented here indicate that, after accounting for the adjustment parameters, no significant relationship of true structural parameters can be found between population, employment, and broadband across 2008 to 2010. This is a key insight as spurious results can be drawn from reduced form estimates if equilibrium adjustments are unaccounted for.

The last Wald test investigates whether the parameters associated with the broadband equation 16c are jointly different from zero. Failure to reject the null hypothesis is evidence that the broadband process may not be endogenous and would reduce the Extended model to the Population-Employment Model. All of the estimators reject the null hypothesis of no relationship except for the FGS3SLS estimator with number of establishments as the employment variable. This gives evidence that broadband is an endogenous variable within the context of migration and employment. Even with broadband as an endogenous process, the structural results between the Extended and Population-Employment models remain the same.

Table 12: Extended Model: Wald Tests for Spatial Parameters

Emp. Variable:	Total Employment			No. of Establishments		
	OLS	GS2SLS	FGS3SLS	OLS	GS2SLS	FGS3SLS
Null Hypothesis						
$\phi_{ea} = 0$	0.0756	0.1747	0.1781	1.2054	3.6506°	0.3207
	(0.5345)	(0.4308)	(0.4282)	(0.1469)	(0.0284)	(0.3451)
$\phi_{ba} = 0$	13.5082•	26.8932•	20.611•	19.2463•	12.1945•	1.6094
	(1e-04)	(0)	(0)	(0)	(2e-04)	(0.1081)
$\phi_{pb} = 0$	0.3335	0.4942	0.5539	0.0068	0.1234	0.1195
	(0.3394)	(0.2803)	(0.2629)	(0.7433)	(0.4759)	(0.48)
$\phi_{bb} = 0$	0.3531	0.1025	0.084	1.7722†	0.016	0.7257
	(0.331)	(0.4988)	(0.5224)	(0.0962)	(0.6828)	(0.2217)
$\phi_{p\Gamma} = 0$	0.0359	0.0191	0	0.4216	0.024	0.0347
	(0.6123)	(0.6685)	(0.9706)	(0.3044)	(0.649)	(0.6155)
$\phi_{e\Gamma} = 0$	0.2849	0.5923	0.3869	0.03	0.0037	0.0007
	(0.3624)	(0.2527)	(0.3173)	(0.6291)	(0.7791)	(0.8555)
$\phi_{pb} = \phi_{p\Gamma} = 0$	0.3694	0.5133	0.5821	0.4285	0.1475	0.1417
	(0.8313)	(0.7736)	(0.7475)	(0.8072)	(0.9289)	(0.9316)
$\phi_{ea} = \phi_{e\Gamma} = 0$	0.3606	0.767	0.5643	1.2354	3.6543	0.321
	(0.835)	(0.6815)	(0.7542)	(0.5392)	(0.1609)	(0.8517)
$\phi_{ba} = \phi_{bb} = 0$	20.2262•	27.3737•	20.8108•	19.2594•	12.22•	1.6099
	(0)	(0)	(0)	(1e-04)	(0.0022)	(0.4471)

P-Values in parenthesis. Significance at the 1%, 5%, and 10% levels indicated by •, °, and †, respectively.

Finally, table 12 presents Wald tests for the spatial parameters corresponding to housing, labor, and broadband markets. The estimated structural relationships fails to indicate a significant relationship for any of these markets with the exception of the broadband market for population, ϕ_{ba}.[23] The algebra behind this structural parameter would indicate roughly a coefficient between -0.886 to -1.195 on the spatially lagged broadband variables for the population equation. Since the regression coefficients are in terms of z-scores, the interpretation would be that an increase of one standard deviation in the average of broadband providers for neighbors of a county (5.1 in 2008 or 1.1 from 2008 to 2010) would lead to a decrease in approximately 0.886 standard deviations of migration from 2008 to 2010 (3,494). Succinctly, if the neighbors of a county, on average, had one more broadband provider each in 2008 then a county can expect a decrease of approximately 607 migrants. This effect can be interpreted as an indication of counties competing against each other within a region for migrants. This is a strong effect, economically, although it is important to emphasize that the effect is if *all* neighbors for a county had one more broadband provider in 2008. Given that the average number of neighbors for a county is 6, the spatial effects would necessitate that the neighbors collectively had 6 more providers on average.

6.3 Robustness Checks

An argument in the evaluation of broadband as a driver for economic growth is that only certain sectors of the economy are impacted by the use of broadband (Kolko, 2012). Because aggregate employment is used in the previous section, it is possible that the effects of broadband may be masked for particular industries. In order to evaluate this claim, employment is broken down by NAICS classification and the Population-Employment Model re-estimated. Table 13 displays the structural parameter $\alpha_{1,BB}$ and its corresponding standard error via the Delta Method (as described in section 4.3.1). Table 14 does the same for $\beta_{1,BB}$. The corresponding p-values for each table are from Wald tests for the structural parameters as in section 6.1.2. As can be seen in the tables, the conclusions drawn from the two equation model do not change as none of the NAICS classifications indicate that broadband causes

[23] This is for all estimators except for the FGS3SLS with number of establishments as the employment variable.

employment or population growth for a county.

Table 13: NAICS Classifications: $\alpha_{1,BB}$

Emp. Variable:	Total Employment			No. of Establishments		
Industry (NAICS Classification)	Delta	S.E.	P-Val	Delta	S.E.	P-Val
All Firms	3.19	(16.439)	0.846	0.649	(1.666)	0.697
Only Private Firms	0.855	(1.346)	0.525	1.42	(4.061)	0.727
Agriculture, Forestry, Fishing and Hunting (11)	0.608	(0.946)	0.52	0.791	(1.864)	0.672
Mining (21)	0.661	(0.734)	0.368	0.472	(1.276)	0.711
Utilities (22)	0.162	(0.684)	0.813	0.0663	(0.209)	0.751
Construction (23)	0.438	(0.914)	0.632	-2.19	(7.208)	0.761
Manufacturing (31-33)	6.48	(31.140)	0.835	-0.0357	(1.699)	0.983
Wholesale Trade (42)	0.22	(2.225)	0.921	0.755	(1.041)	0.468
Retail Trade (44-45)	-0.0979	(0.246)	0.691	-0.671	(6.381)	0.916
Transportation and Warehousing (48-49)	0.689	(2.322)	0.767	0.142	(0.432)	0.742
Information (51)	-0.62	(1.278)	0.627	2	(3.951)	0.613
Finance and Insurance (52)	-0.051	(1.014)	0.96	2.16	(52.263)	0.967
Real Estate and Rental and Leasing (53)	0.285	(1.110)	0.797	0.591	(2.609)	0.821
Professional, Scientific, and Technical Services (54)	0.497	(0.967)	0.607	-0.00825	(1.576)	0.996
Management of Companies and Enterprises (55)	0.47	(0.641)	0.464	0.0199	(0.156)	0.899
Administrative and Business Support Services (56)	0.0317	(0.161)	0.844	0.462	(2.065)	0.823
Educational Services (61)	0.362	(1.899)	0.849	-0.832	(4.141)	0.841
Health Care and Social Assistance (62)	0.29	(0.566)	0.608	-0.218	(5.704)	0.97
Arts, Entertainment, and Recreation (71)	0.147	(1.784)	0.934	-101	(31,270.207)	0.997
Accommodation and Food Services (72)	0.415	(0.445)	0.35	-2.15	(12.730)	0.866
Other Services (except Public Administration) (81)	-0.474	(7.066)	0.946	0.367	(0.666)	0.582

Table 14: NAICS Classifications: $\beta_{1,BB}$

Emp. Variable:	Total Employment			No. of Establishments		
Industry (NAICS Classification)	Delta	S.E.	P-Val	Delta	S.E.	P-Val
All Firms	0.0868	(0.133)	0.515	0.459	(1.266)	0.717
Only Private Firms	0.179	(1.177)	0.879	0.292	(0.842)	0.729
Agriculture, Forestry, Fishing and Hunting (11)	1.49	(2.828)	0.6	0.63	(18.703)	0.973
Mining (21)	0.29	(1.673)	0.862	-0.435	(2.093)	0.835
Utilities (22)	-0.189	(0.880)	0.83	-0.0716	(0.420)	0.865
Construction (23)	0.272	(0.655)	0.678	0.149	(0.263)	0.572
Manufacturing (31-33)	-0.113	(0.313)	0.718	0.0116	(0.120)	0.924
Wholesale Trade (42)	-0.0667	(0.245)	0.785	-0.683	(0.852)	0.423
Retail Trade (44-45)	0.0296	(0.126)	0.814	0.505	(0.769)	0.511
Transportation and Warehousing (48-49)	-0.28	(7.322)	0.97	-0.984	(5.629)	0.861
Information (51)	-0.309	(1.579)	0.845	-0.0945	(0.490)	0.847
Finance and Insurance (52)	0.0219	(0.148)	0.882	-0.0351	(0.276)	0.899
Real Estate and Rental and Leasing (53)	0.402	(0.590)	0.496	0.494	(0.911)	0.587
Professional, Scientific, and Technical Services (54)	0.12	(0.257)	0.64	0.0237	(0.620)	0.969
Management of Companies and Enterprises (55)	-0.582	(1.179)	0.622	-0.00618	(0.083)	0.941
Administrative and Business Support Services (56)	0.0167	(0.068)	0.806	-4.48	(48.148)	0.926
Educational Services (61)	0.492	(6.674)	0.941	0.0989	(0.141)	0.482
Health Care and Social Assistance (62)	0.281	(0.632)	0.657	0.0454	(2.865)	0.987
Arts, Entertainment, and Recreation (71)	0.195	(4.297)	0.964	0.408	(1.131)	0.718
Accommodation and Food Services (72)	-0.154	(0.908)	0.865	0.3	(0.708)	0.672
Other Services (except Public Administration) (81)	-12.2	(450.580)	0.978	0.227	(0.171)	0.184

7 Concluding Remarks

In this paper, I have developed a framework for evaluating three inter-related processes: population growth, employment growth, and broadband diffusion. Employing a structural approach such as has been used here is necessary because a reduced form framework may obscure how each of these processes influences the others. Indeed, with a structural model involving population, employment, and broadband I demonstrate that while the reduced form estimates indicate significant relationships among the three processes, the structural parameters do not find any significant relationships. The adjustment parameter of a model involving dynamic variables that, on their own, converge towards levels independent of other factors is the key element in a structural relationship between population, employment, and broadband that temper the reduced form findings.

For broadband, this paper casts doubt on claims that broadband can be a driver of economic growth in the United States. The structural relationship does not appear to indicate that "jobs follow broadband" or that "people follow broadband" in order to justify claims that infrastructure development in broadband for a county or municipality will be a driver in growth. This paper does indicate how one might be persuaded to think that broadband can drive economic growth as the reduced form estimates in the Extended Model for Broadband would indicate that "jobs follow broadband" when the endogeneity of the broadband process is accounted for.

There is cause for concern as to how flexible the structural model is. If the structural model is not a true reflection of how the three processes develop, then the estimated structural parameters may not be informative. Determining the flexibility of the model to different functional forms for each of the processes is left for future research to consider other counterfactual models in assessing how broadband can impact an economy.

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A Appendix

A.1 Dynamic System

In order to have a stable equilibrium for the system, the constraints $\lambda_P, \lambda_E \in (0,1)$ need to be imposed. This can be seen by rearranging equation 7 in terms of a linear partial difference equation in $P_{i,t}, P_{i,t-1}, E_{i,t}$ and $E_{i,t-1}$:

$$\begin{bmatrix} P_{i,t} \\ E_{i,t} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} P_{i,t-1} \\ E_{i,t-1} \end{bmatrix} + \begin{bmatrix} B_{P,0} \\ B_{E,0} \end{bmatrix} \tag{41}$$

Where $B_{P,0}$ and $B_{E,0}$ are vectors of initial conditions of the exogenous variables. The coefficients for A_{11}, A_{12}, A_{21} and A_{22} are functions of λ_P and λ_E. For the system to be stable, the eigenvalues of the coefficient matrix needs to be strictly less than 1 in absolute value. The implication here is that λ_P and λ_E can be estimated in an unrestricted regression, and then tested to see if their estimates imply a stable equilibrium. If they do not, I can re-estimate the equation with a restriction on the equilibrium adjustment parameters to force a stable system. This becomes an empirical question of the validity of the model.

A.1.1 Reduced Form: Population-Employment Model

The reduced form models of 8 can be expressed by rearranging and grouping terms as:

$$P_{i,t} = A_1 Z + \Xi_2 E_{i,t} + \Xi_3 E_{i,t-1} + (1-\lambda_P) P_{i,t-1} \tag{42a}$$

$$E_{i,t} = B_1 Z + \Psi_2 P_{i,t} + \Psi_3 P_{i,t-1} + (1-\lambda_E) E_{i,t-1} \tag{42b}$$

where $\Xi_2 = (I + \phi_E W_E) \frac{\alpha_2 \lambda_P}{\lambda_E}$, $\Xi_3 = (I + \phi_E W_E) \frac{\alpha_2 \lambda_P (\lambda_E - 1)}{\lambda_E}$, $\Psi_2 = (I + \phi_P W_P) \frac{\beta_2 \lambda_E}{\lambda_P}$, and $\Psi_3 = (I + \phi_P W_P) \frac{\beta_2 \lambda_E (\lambda_P - 1)}{\lambda_P}$. In order to evaluate the stability of the system, both $P_{i,t}$ and $E_{i,t}$ need to be expressed as a function of exogenous variables.

Substituting $E_{i,t}$ from 42b into equation 42a gives us:

$$P_{i,t} = A_1 Z + \Xi_2 \big[\mathbf{B_1 Z} + \mathbf{\Psi_2 P_{i,t}} + \mathbf{\Psi_3 P_{i,t-1}} + \left(\mathbf{1} - \lambda_{\mathbf{E}}\right) \mathbf{E_{i,t-1}} \big] + \Xi_3 E_{i,t-1} + \left(1 - \lambda_P\right) P_{i,t-1} \quad (43a)$$

$$\left(I - \Xi_2 \Psi_2\right) P_{i,t} = \left(A_1 \Xi_2 B_1\right) Z + \left(1 - \lambda_P + \Xi_2 \Psi_3\right) P_{i,t-1} + \big(\Xi_2 \left(1 - \lambda_E\right) + \Xi_3\big) E_{i,t-1} \quad (43b)$$

$$P_{i,t} = G^{-1} \left(A_1 \Xi_2 B_1\right) Z + G^{-1} \left(1 - \lambda_P + \Xi_2 \Psi_3\right) P_{i,t-1} + G^{-1} \left(\Xi_2 \left(1 - \lambda_E\right) + \Xi_3\right) E_{i,t-1} \quad (43c)$$

where $G = \left(I - \Xi_2 \Psi_2\right)$. With $P_{i,t}$ solved in terms of exogenous variables, equation 43c can be substituted into equation 42b:

$$E_{i,t} = B_1 Z + \Psi_2 \big[\mathbf{G^{-1}} \left(\mathbf{A_1 \Xi_2 B_1}\right) \mathbf{Z} + \mathbf{G^{-1}} \left(\mathbf{1} - \lambda_{\mathbf{P}} + \mathbf{\Xi_2 \Psi_3}\right) \mathbf{P_{i,t-1}} + \mathbf{G^{-1}} \left(\mathbf{\Xi_2} \left(\mathbf{1} - \lambda_{\mathbf{E}}\right) + \mathbf{\Xi_3}\right) \mathbf{E_{i,t-1}} \big] + \Psi_3 P_{i,t-1} + \left(1 - \lambda_E\right) E_{i,t-1} \quad (44a)$$

$$E_{i,t} = \big(B_1 + \Psi_2 G^{-1} \left(A_1 \Xi_2 B_1\right)\big) Z + \big(\Psi_3 + \Psi_2 G^{-1} \left(1 - \lambda_P + \Xi_2 \Psi_3\right)\big) P_{i,t-1} + \big(1 - \lambda_E + \Psi_2 G^{-1} \left(\Xi_2 \left(1 - \lambda_E\right) + \Xi_3\right)\big) E_{i,t-1} \quad (44b)$$

With equations 43c and 44b as the simplified versions of the model, the system can be put into the form of equation 41. In order to do this, the terms involving a spatial weight matrix (W_E and W_P) need to be evaluated as an average spatial effect. The system would be highly non-linear due to the non-zero values of a spatial weight matrix which would be computationally infeasible. The above equations can be rearranged in terms of exogenous variables as follows:

$$\begin{bmatrix} P_{i,t} \\ E_{i,t} \end{bmatrix} = \begin{bmatrix} 1-\lambda_P & 0 \\ 0 & 1-\lambda_E \end{bmatrix} \begin{bmatrix} P_{i,t-1} \\ E_{i,t-1} \end{bmatrix} + \begin{bmatrix} B_{P,0} \\ B_{E,0} \end{bmatrix} \tag{45}$$

where $B_{P,0}$ and $B_{E,0}$ are functions of exogenous variables. The condition for a stable equilibrium is that the largest eigenvalues of the matrix above must be less than 1 in absolute value. This is why the constraints $\lambda_P, \lambda_E \in (0,1)$ need to be imposed.

A.2 Three Equation

For the Extended Model for Broadband, there are six parameters of interest: α_1, α_2, β_1, β_2, γ_1, and γ_2. Similar to the process of evaluating these parameters using the Delta Method, as in section 4.3.1, I can stack the parameters of interest into vector form as follows:

$$\delta = \begin{bmatrix} A_1 \\ A_2 \\ \Phi_{E,A} \\ \frac{A_2}{\lambda_E} \\ \frac{\Phi_{E,A}}{\lambda_E} \\ A_3 \\ \Phi_{B,A} \\ \frac{A_3}{\lambda_B} \\ \frac{\Phi_{B,A}}{\lambda_B} \\ -\lambda_P \\ B_1 \\ B_2 \\ \Phi_{P,B} \\ \frac{B_2}{\lambda_P} \\ \frac{\Phi_{P,B}}{\lambda_P} \\ B_3 \\ \Phi_{B,B} \\ \frac{B_3}{\lambda_B} \\ \frac{\Phi_{B,B}}{\lambda_B} \\ -\lambda_E \\ \Gamma_1 \\ \Gamma_2 \\ \Phi_{P,\Gamma} \\ \frac{\Gamma_2}{\lambda_P} \\ \frac{\Phi_{P,\Gamma}}{\lambda_P} \\ \Gamma_3 \\ \Phi_{E,\Gamma} \\ \frac{\Gamma_3}{\lambda_E} \\ \frac{\Phi_{E,\Gamma}}{\lambda_E} \\ -\lambda_B \end{bmatrix} = \begin{bmatrix} \alpha_1\lambda_P \\ \alpha_2\lambda_P \\ \alpha_2\lambda_P\phi_{EA} \\ \frac{\alpha_2\lambda_P}{\lambda_E} \\ \frac{\alpha_2\lambda_P\phi_{EA}}{\lambda_E} \\ \alpha_3\lambda_P \\ \alpha_3\lambda_P\phi_{BA} \\ \frac{\alpha_3\lambda_P}{\lambda_B} \\ \frac{\alpha_3\lambda_P\phi_{BA}}{\lambda_B} \\ -\lambda_P \\ \beta_1\lambda_E \\ \beta_2\lambda_E \\ \beta_2\lambda_E\phi_{PB} \\ \frac{\beta_2\lambda_E}{\lambda_P} \\ \frac{\beta_3\lambda_E\phi_{BB}}{\lambda_P} \\ \beta_3\lambda_E \\ \beta_3\lambda_E\phi_{BB} \\ \frac{\beta_3\lambda_E}{\lambda_B} \\ \frac{\beta_3\lambda_E\phi_{BB}}{\lambda_B} \\ -\lambda_E \\ \gamma_1\lambda_B \\ \gamma_2\lambda_B \\ \gamma_2\lambda_B\phi_{P\Gamma} \\ \frac{\gamma_2\lambda_B}{\lambda_P} \\ \frac{\gamma_2\lambda_B\phi_{P\Gamma}}{\lambda_P} \\ \gamma_3\lambda_B \\ \gamma_3\lambda_B\phi_E \\ \frac{\gamma_3\lambda_B}{\lambda_E} \\ \frac{\gamma_3\lambda_B\phi_{E\Gamma}}{\lambda_E} \\ -\lambda_B \end{bmatrix} \tag{46}$$

From the stacked vector of parameters, I can construct a function of the reduced form paramteres in order to recover the structural parameters of interest. As seen with 4.3.1, I can let $r(\cdot)$ be a function of the parameters in the model:

$$r_2(\delta) = \begin{bmatrix} \frac{A_2}{\lambda_P} \\ \frac{A_3}{\lambda_P} \\ \frac{B_2}{\lambda_E} \\ \frac{B_3}{\lambda_E} \\ \frac{\Gamma_2}{\lambda_B} \\ \frac{\Gamma_3}{\lambda_B} \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ \alpha_3 \\ \beta_2 \\ \beta_3 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \tag{47}$$

where I would like to evaluate each of these restrictions one at a time as well as across equations. This can be done by constructing a Wald test at which I assume the true parameter δ and take a Taylor series expansion about this value. This allows for inference upon the null hypothesis of $H_0 : r(\delta) = 0$ This involves taking the derivative of the restriction vector in equation 36 with respect to the true parameters in δ:

$$R_2(\delta) \equiv \frac{\partial r_2(\delta)}{\partial \delta'} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{\lambda_P} & 0 & 0 & 0 & 0 & 0 \\
\frac{A_2}{\lambda_P \Phi_{E,A}} & 0 & 0 & 0 & 0 & 0 \\
\frac{\lambda_E}{\lambda_P} & 0 & \frac{B_2}{A_2} & \frac{B_3}{A_2} & 0 & 0 \\
\frac{A_2 \lambda_E}{\lambda_P \Phi_{E,A}} & 0 & \frac{B_2}{\Phi_{E,A}} & \frac{B_3}{\Phi_{E,A}} & 0 & 0 \\
0 & \frac{1}{\lambda_P} & 0 & 0 & 0 & 0 \\
0 & \frac{A_3}{\lambda_P \Phi_{B,A}} & 0 & 0 & 0 & 0 \\
0 & \frac{\lambda_B}{\lambda_P} & 0 & 0 & \frac{\Gamma_2}{A_3} & \frac{\Gamma_3}{A_3} \\
0 & \frac{A_3 \lambda_B}{\lambda_P \Phi_{B,A}} & 0 & 0 & \frac{\Gamma_2}{\Phi_{B,A}} & \frac{\Gamma_3}{\Phi_{B,A}} \\
\frac{A_2}{\lambda_P^2} & \frac{A_3}{\lambda_P^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\lambda_E} & 0 & 0 & 0 \\
0 & 0 & \frac{B_2}{\lambda_E \Phi_{P,B}} & 0 & 0 & 0 \\
\frac{A_2}{B_2} & \frac{A_3}{B_2} & \frac{\lambda_P}{\lambda_E} & 0 & 0 & 0 \\
\frac{A_2}{\Phi_{P,B}} & \frac{A_3}{\Phi_{P,B}} & \frac{B_2 \lambda_P}{\lambda_E \Phi_{P,B}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\lambda_E} & 0 & 0 \\
0 & 0 & 0 & \frac{B_3}{\lambda_E \Phi_{B,B}} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_B}{\lambda_E} & \frac{\Gamma_2}{B_3} & \frac{\Gamma_3}{B_3} \\
0 & 0 & 0 & \frac{B_3 \lambda_B}{\lambda_E \Phi_{B,B}} & \frac{\Gamma_2}{\Phi_{B,B}} & \frac{\Gamma_3}{\Phi_{B,B}} \\
0 & 0 & \frac{B_2}{\lambda_E^2} & \frac{B_3}{\lambda_E^2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\lambda_B} & 0 \\
0 & 0 & 0 & 0 & \frac{\Gamma_2}{\lambda_B \Phi_{P,\Gamma}} & 0 \\
\frac{A_2}{\Gamma_2} & \frac{A_3}{\Gamma_2} & 0 & 0 & \frac{\lambda_P}{\lambda_B} & 0 \\
\frac{A_2}{\Phi_{P,\Gamma}} & \frac{A_3}{\Phi_{P,\Gamma}} & 0 & 0 & \frac{\Gamma_2 \lambda_P}{\lambda_B \Phi_{P,\Gamma}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\lambda_B} \\
0 & 0 & 0 & 0 & 0 & \frac{\Gamma_3}{\lambda_B \Phi_{E,\Gamma}} \\
0 & 0 & \frac{B_2}{\Gamma_3} & \frac{B_3}{\Gamma_3} & 0 & \frac{\lambda_E}{\lambda_B} \\
0 & 0 & \frac{B_2}{\Phi_{E,\Gamma}} & \frac{B_3}{\Phi_{E,\Gamma}} & 0 & \frac{\Gamma_3 \lambda_E}{\lambda_B \Phi_{E,\Gamma}} \\
0 & 0 & 0 & 0 & \frac{\Gamma_2}{\lambda_B^2} & \frac{\Gamma_3}{\lambda_B^2}
\end{bmatrix}' \tag{48}$$

The Wald Tests constructed from equation 48 allows for inference upon the relationship between broadband and jobs/people.

One potential problem with the wald tests associated with equation 48 is that the structural three equation model implies that the broadband deployment process is endogenous. The endogeneity of broadband can be formulated as an empirical question. If broadband deployment is an exogenous process, then the causal inferences drawn from the above Wald Tests are not valid. To determine the validity of a three equation model, the following restriction on structural paramters is tested:

$$\gamma_1 = \gamma_2 = \gamma_3 = \phi_{BA} = \phi_{BB} = \alpha_3 = \beta_3 = 0 \tag{49}$$

Of note with the above restriction is that the equilibrium adjustment parameter, λ_B, is not present. This is because the model becomes undefined if $\lambda_B = 0$ is imposed due to the interaction with the equilibrium adjustment and all other parameters in the model. This particular restriction is across multiple parameters and can be constructed via a restriction vector of the following:

$$r_3(\delta) = \begin{bmatrix} \frac{\Gamma_1}{\lambda_B} \\ \frac{\Gamma_2}{\lambda_B} \\ \frac{\Gamma_3}{\lambda_B} \\ \frac{\Phi_{BA}}{A_3} \\ \frac{\Phi_{BB}}{B_3} \\ \frac{A_3}{\lambda_P} \\ \frac{B_3}{\lambda_E} \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \phi_{BA} \\ \phi_{BB} \\ \alpha_3 \\ \beta_3 \end{bmatrix} \tag{50}$$

This leads to a restriction matrix of the following form:

$$
R_3(\delta) \equiv \frac{\partial r_3(\delta)}{\partial \delta'} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{B_3}{A_2} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{B_3}{\Phi_{EA}} \\
0 & 0 & 0 & -\frac{\Phi_{BA}}{A_3^2} & 0 & \frac{1}{\lambda_P} & 0 \\
0 & 0 & 0 & \frac{1}{A_3} & 0 & \frac{A_3}{\lambda_P \Phi_{BA}} & 0 \\
\frac{\Gamma_1}{A_3} & \frac{\Gamma_2}{A_3} & \frac{\Gamma_3}{A_3} & -\frac{\Phi_{BA}\lambda_B}{A_3^2} & 0 & \frac{\lambda_B}{\lambda_P} & 0 \\
\frac{\Gamma_1}{\Phi_{BA}} & \frac{\Gamma_2}{\Phi_{BA}} & \frac{\Gamma_3}{\Phi_{BA}} & \frac{\lambda_B}{A_3} & 0 & \frac{A_3\lambda_B}{\lambda_P \Phi_{BA}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{A_3}{\lambda_P^2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{A_3}{B_2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{A_3}{\Phi_{EB}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\lambda_E} \\
0 & 0 & 0 & 0 & -\frac{\Phi_{BB}}{B_3^2} & 0 & \frac{B_3}{\lambda_E \Phi_{BB}} \\
\frac{\Gamma_1}{B_3} & \frac{\Gamma_2}{B_3} & \frac{\Gamma_3}{B_3} & 0 & \frac{1}{B_3} & 0 & \frac{\lambda_B}{\lambda_E} \\
\frac{\Gamma_1}{\Phi_{BB}} & \frac{\Gamma_2}{\Phi_{BB}} & \frac{\Gamma_3}{\Phi_{BB}} & 0 & -\frac{\Phi_{BB}\lambda_B}{B_3^2} & 0 & \frac{B_3\lambda_B}{\lambda_E \Phi_{BB}} \\
0 & 0 & 0 & 0 & \frac{\lambda_B}{B_3} & 0 & \frac{B_3}{\lambda_E^2} \\
\frac{1}{\lambda_B} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\lambda_B} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\Gamma_2}{\lambda_B \Phi_{P\Gamma}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\lambda_P}{\lambda_B} & 0 & 0 & 0 & \frac{A_3}{\Gamma_2} & 0 \\
0 & \frac{\Gamma_2\lambda_P}{\lambda_B \Phi_{P\Gamma}} & 0 & 0 & 0 & \frac{A_3}{\Phi_{P\Gamma}} & 0 \\
0 & 0 & \frac{1}{\lambda_B} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\Gamma_3}{\lambda_B \Phi_{E\Gamma}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_E}{\lambda_B} & 0 & 0 & 0 & \frac{B_3}{\Gamma_3} \\
0 & 0 & \frac{\Gamma_3\lambda_E}{\lambda_B \Phi_{E\Gamma}} & 0 & 0 & 0 & \frac{B_3}{\Phi_{E\Gamma}} \\
\frac{\Gamma_1}{\lambda_B^2} & \frac{\Gamma_2}{\lambda_B^2} & \frac{\Gamma_3}{\lambda_B^2} & 0 & 0 & 0 & 0
\end{bmatrix}' \tag{51}
$$

A.2.1 Spatial Parameters

Finally, the three equation model has spatial parameters of interest similar to the two equation model. The structural parameters ϕ_{EA}, ϕ_{BA}, ϕ_{PB}, ϕ_{BB}, $\phi_{P\Gamma}$, and $\phi_{E\Gamma}$ determine the spatial processes associated with the interrelationship of population, employment, and broadband.

$$r_4(\delta) = \begin{bmatrix} \frac{\Phi_{EA}}{A_2} \\ \frac{\Phi_{BA}}{A_3} \\ \frac{\Phi_{PB}}{B_2} \\ \frac{\Phi_{BB}}{B_3} \\ \frac{\Phi_{P\Gamma}}{\Gamma_2} \\ \frac{\Phi_{E\Gamma}}{\Gamma_3} \end{bmatrix} = \begin{bmatrix} \phi_{EA} \\ \phi_{BA} \\ \phi_{PB} \\ \phi_{BB} \\ \phi_{P\Gamma} \\ \phi_{E\Gamma} \end{bmatrix} \tag{52}$$

Taking the derivative of the restriction vector in equation 52 with respect to δ' in equation 46:

$$R_4(\delta) \equiv \frac{\partial r_4(\delta)}{\partial \delta'} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\Phi_{EA}}{A_2^2} & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{A_2} & 0 & 0 & 0 & 0 & 0 \\
-\frac{\Phi_{EA}\lambda_E}{A_2^2} & 0 & 0 & 0 & 0 & 0 \\
\frac{\lambda_E}{A_2} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\Phi_{BA}}{A_3^2} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{A_3} & 0 & 0 & 0 & 0 \\
0 & -\frac{\Phi_{BA}\lambda_B}{A_3^2} & 0 & 0 & 0 & 0 \\
0 & \frac{\lambda_B}{A_3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\Phi_{PB}}{B_2^2} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{B_2} & 0 & 0 & 0 \\
0 & 0 & -\frac{\Phi_{PB}\lambda_P}{B_2^2} & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_P}{B_2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\Phi_{BB}}{B_3^2} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{B_3} & 0 & 0 \\
0 & 0 & 0 & -\frac{\Phi_{BB}\lambda_B}{B_3^2} & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_B}{B_3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{\Phi_{P\Gamma}}{\Gamma_2^2} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\Gamma_2} & 0 \\
0 & 0 & 0 & 0 & -\frac{\Phi_{P\Gamma}\lambda_P}{\Gamma_2^2} & 0 \\
0 & 0 & 0 & 0 & \frac{\lambda_P}{\Gamma_2} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\Phi_{E\Gamma}}{\Gamma_3^2} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\Gamma_3} \\
0 & 0 & 0 & 0 & 0 & -\frac{\Phi_{E\Gamma}\lambda_E}{\Gamma_3^2} \\
0 & 0 & 0 & 0 & 0 & \frac{\lambda_E}{\Gamma_3} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}' \tag{53}$$